

scores on Test 3 for different ability groups among the 157 pairs used to generate Table 11. However, this discussion should be treated with caution. The fact that the Treatment-by-Ability interaction detected in the analysis of these 157 tests was not confirmed in the analysis of all 265 pairs of students for whom some ability score was available raises the possibility that the interaction could be spurious.

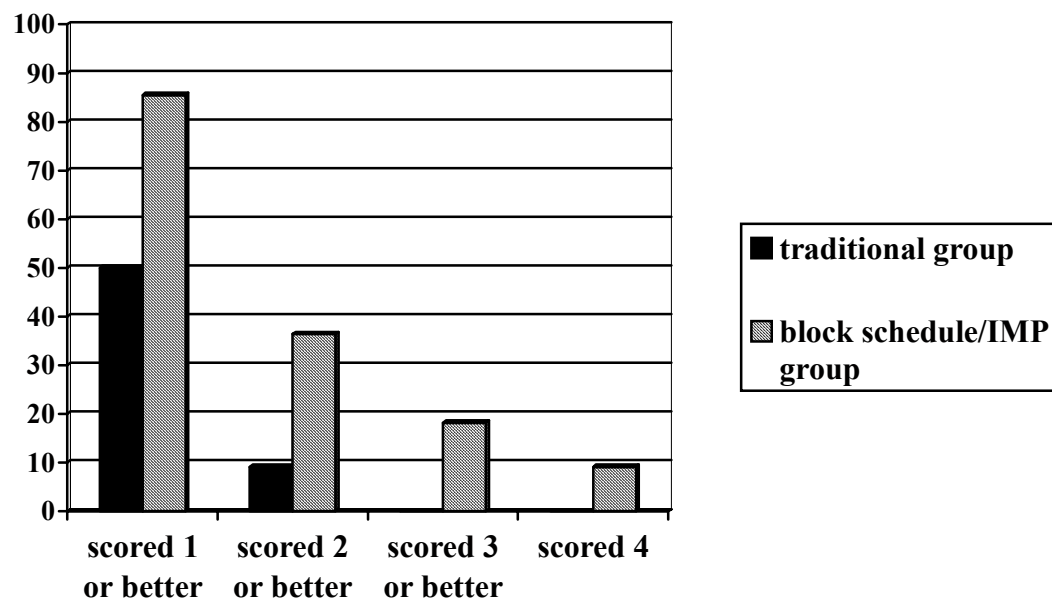
To analyze possibly differential effects of Treatment based on pair Ability, the 157 pairs with two Grade 6 Quantitative Ability scores were split into two groups. As noted above, these 157 pairs included 44 pairs in the Traditional group. Of the 44 Traditional group pairs, 22 had scores on zAbility below 0.13 and 22 had scores on zAbility above 0.13. Because there were a smaller number of students in the Traditional group than in the Reform, 0.13 was chosen as a cut-point to split the 157 tests into a “Lower Ability” group and a “Higher Ability” group.

The Lower Ability group contained 77 pairs. These included 22 pairs from the Traditional cohort, 26 pairs from the First Reform cohort, and 29 pairs from the Second Reform cohort. For these pairs, a Probit Analysis was performed using Test 3 score as a dependent variable and an indicator of Reform group vs. Traditional group as an independent variable. On average, Lower Ability pairs in the Reform cohorts had an estimated achievement that was 1.07 standard deviations higher than that of Lower Ability pairs in the Traditional cohort. This difference was significant (Wald Chi-square with 1 degree of freedom =13.01, $p=.0003$). Assuming that achievement is normally distributed, then if a pair would ordinarily score at the 50th percentile, improving their score by 1.07 standard deviations would move the pair up to the 86th percentile. A 95% confidence interval indicates that for these low-ability pairs the advantage of the Reform

group was between +0.49 and +1.65 standard deviations. A Bonferroni-adjusted 98.3% confidence interval indicates that the advantage of the Reform group was probably between +0.36 and +1.78 standard deviations.

Alternatively, a Logistic Regression Analysis determined that among these Lower Ability pairs being in a Reform cohort increased the odds of getting a high score by a factor of 5.99, significant at the .001 level. If the probability of an event is in the middle range, increasing the odds of success by a factor of 5.99 increases the event probability by about 36%. Figure 1 shows that for low-ability students, being in a Reform cohort increased the odds of getting a high score in about this way.

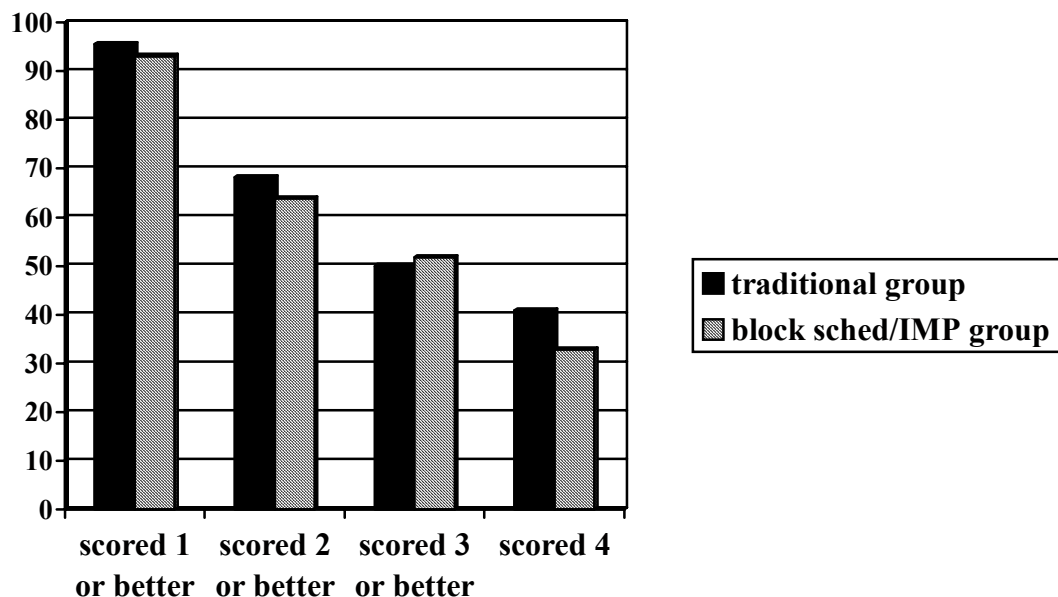
Figure 1. Low ability pairs: Percent scoring at each level on Test 3, applied algebra task.



The Higher Ability group contained 80 pairs. These included 22 pairs from the Traditional cohort, 28 pairs from the First Reform cohort, and 30 pairs from the Second Reform cohort. For these pairs, a Probit Analysis was performed using Test 3 score as a dependent variable and an indicator of Reform group vs. Traditional group as an independent variable. On average, Higher Ability pairs in the Reform cohorts had an estimated achievement that was 0.14 standard deviations lower than that of Higher Ability pairs in the Traditional cohort. This difference was not significant (Wald Chi-square with 1 degree of freedom = 0.273, $p > .5$). Alternatively, a Logistic Regression Analysis indicated that for these Higher Ability pairs membership in a Reform cohort decreased the odds of receiving a high score by a non-significant factor of 1.23 ($p > .5$). In the middle ranges, this would decrease the probability of getting a high score by about 5%. Figure 2 displays achievement on Test 3 for Higher Ability pairs in the Traditional cohort and Higher Ability pairs in the Reform cohorts.

Neither the statistical analysis nor Figure 2 showed any important difference between the Traditional and Reform cohorts in the achievement of pairs in the top 50% of prior ability on an applied algebra task. But could the interaction between Ability and Treatment indicate that in the Reform cohorts the achievement of pairs having the very highest ability suffered? Apparently not. A Probit analysis looked at the achievement of pairs in the top 25%: those with $z\text{Ability}$ greater than 0.80. This “Very High Ability” group contained 34 pairs. These included 11 pairs from the Traditional cohort, 9 pairs from the First Reform cohort, and 14 pairs from the Second Reform cohort. Results indicated that differences were non-significant ($p > .50$), but favored the Reform cohorts, indicating that on average Very High Ability pairs in the Reform cohorts had an

Figure 2. High ability pairs: Percent scoring at each level on Test 3, applied algebra task.



estimated achievement that was 0.16 standard deviations higher than that of Very High Ability pairs in the Traditional cohort.

Test 3: Opposite interaction pattern for pairs with one Grade 6 ability estimate.

As noted previously, no significant Ability-by-Treatment interaction was detected when analyzing the set of 265 tests with at least one Grade 6 Quantitative Ability score available. Given the strength of the interaction pattern for the 157 pairs for whom both students had Grade 6 Quantitative Ability scores, something very different appears to have been going on for the remaining 108 tests.

In fact, for these tests the pattern of interaction is non-significant, but appears to go in the opposite direction. Of the 108 pairs of students with only one Grade 6 Quantitative Ability score, 64 were Lower Ability, with a z_{Ability} less than 0.13. These included 23 pairs from the Traditional cohort, 25 pairs from the First Reform cohort, and 16 pairs from the Second Reform cohort. Among these students, a Probit Analysis indicated almost no difference between pairs from the Traditional cohort and pairs from the Reform cohorts, with a non-significant effect size of 0.01 standard deviations in favor of pairs in the Reform cohorts.

Of the 108 pairs of students with only one Grade 6 Quantitative Ability score, 44 had Higher Ability, with a z_{Ability} greater than 0.13. These included 17 pairs from the Traditional cohort, 13 pairs from the First Reform cohort, and 14 pairs from the Second Reform cohort. Among these students, a Probit Analysis indicated that pairs from the Reform cohorts outscored pairs from the Traditional cohort by 0.31 standard deviations. This difference was not statistically significant (Wald Chi-square=.878 with 1 degree of

freedom, $p=.349$), but the magnitude of the effect was greater than the 0.28 standard deviations reported for the full sample of 290 tests.

Test 3 interactions: A possible interpretation. Given the contrasting results on tests for a treatment-by-ability interaction for Test 3, depending on how “ability” was measured, it is quite possible that the interaction detected was spurious. The lack of any indication that there might be a treatment-by-ability interaction for Test 1 or Test 2 lends weight to the theory that the possible interaction for Test 3 was spurious.

If, however, the Test 3 interaction reflected a real pattern in student algebra learning, one possible explanation presents itself. Most often, students with Grade 6 Quantitative Ability scores were those who attended Suburban High School’s feeder middle school and used the *Visual Mathematics* curriculum throughout Grades 6, 7, and 8. Students lacking Grade 6 Quantitative Ability scores were those who had not done so. The very different interaction patterns for pairs with two Grade 6 Quantitative Ability scores, vs. pairs with only one Grade 6 Quantitative Ability score, raises the possibility of a synergistic effect between the *Visual Mathematics* curriculum and the IMP/block schedule treatment.

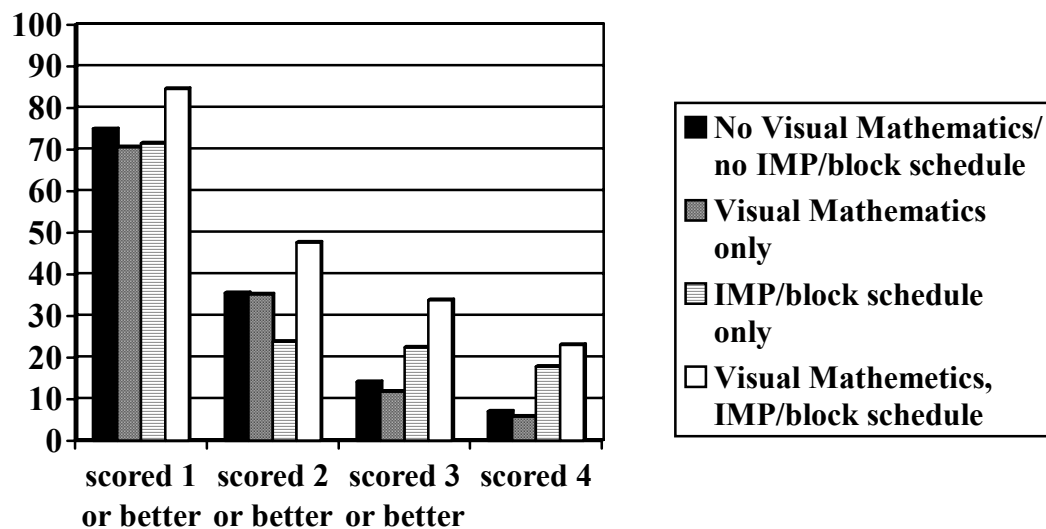
The data are consistent with the following explanation: Perhaps low-ability students were helped to solve problems comparable to the item in Test 3 if they used both the *Visual Mathematics* curriculum and the block schedule/IMP curriculum, but anything less was not sufficient. In contrast, perhaps high-ability students were helped to solve problems such as that presented in Test 3 if they used either the *Visual Mathematics* curriculum or the block schedule/IMP curriculum. Both experiences did not further enhance their achievement beyond that attributed to either curriculum.

The Suburban High School transcript files had Grade 6 transcripts available for students in the Traditional and First Reform cohorts, but not for students in the Second Reform cohort. For students with Grade 6 transcripts available, it was possible to obtain confirmatory information about whether they were enrolled in the middle school program that used *Visual Mathematics*. Too few pairs with sixth-grade ability estimates were available for this study to investigate how *Visual Mathematics* and the IMP/block schedule treatment interacted for high versus low ability groups, but it was possible to perform a tentative investigation of whether there was an interaction overall, ignoring student ability.

In Figure 3 pairs were coded as “No Visual Mathematics, No IMP/block schedule” if they were in the Traditional cohort and at least one member of the pair was lacking both a Grade 6 transcript and Grade 6 ability scores (28 pairs). Pairs were coded as “Visual Mathematics only” if they were in the Traditional cohort and both members of the pair had either a Grade 6 transcript or Grade 6 ability scores (67 pairs). Pairs were coded as “IMP/block schedule only” if they were in the First Reform cohort and at least one member was lacking both a Grade 6 transcript and Grade 6 ability scores (34 pairs). Pairs were coded as “Visual Mathematics, IMP/block schedule” if they were in the First Reform cohort and both members of the pair had either a Grade 6 transcript or Grade 6 ability scores (65 pairs). Figure 3 shows that pairs in which both members had used the *Visual Mathematics* curriculum, who furthermore were in the cohort using a block schedule and IMP, tended to outscore all other groups.

A Logistic Regression Analysis was run to see if this apparent difference could be dismissed as purely chance. Pairs who were in the First Reform *and* for whom both members had used the *Visual Mathematics* curriculum were compared to all other pairs in the Traditional or First Reform cohorts. Being in the “*Visual Math* plus IMP/block” group multiplied the odds of getting a high score by a factor of 1.91, significant at the .019 level. If the probability of an event is in the middle range, increasing the odds of success by a factor of 1.91 increases the event probability by about 16%.

Figure 3. Four groups: Percent scoring at each level on Test 3, applied algebra task.



The statistical significance level of 0.019 should be viewed as an indication that the pattern evident in Figure 3 cannot be dismissed as purely random. Nonetheless the reader should be cautious about giving the significance level too much weight. A 0.019 significance level does not meet the Bonferroni-adjusted 0.017 level used by this study. More importantly, the comparison of “*Visual Math* plus IMP/block schedule” to other groups was not among the pre-planned significance tests for this study and was made after looking at the data in order to explain an unusual interaction pattern. Any data set will have some patterns that occur by chance, and looking for patterns followed by statistical significance testing will often obtain spuriously “significant” results. Thus, the possibility of a synergistic effect between the *Visual Mathematics* curriculum and the IMP/block schedule must be viewed as an interesting possibility raised by this study, not as a confirmed hypothesis.

Specific Effects of Traditional Curriculum/Schedule

and IMP Curriculum/Block Schedule

This section provides details about the performance of the Traditional cohort and the Reform cohorts on specific questions or subsets of questions. The details illuminate the degree to which students in each group appear to have mastered specific skills underlying their scores on Test 1 and Test 2.

The discussion below generally reports “average score” on individual items. Because partial credit was given if, for example, a student made a minor arithmetic error but otherwise showed correct work, a student’s score was computed as that student’s score on the problem(s), divided by maximum possible score. See the scoring rubrics in Appendix B for details on how credit was assigned for each problem.

On individual items that were scored as right/wrong, a Pearson Chi-square statistic was computed from a cross-tabulation table. On items for which students could receive partial credit, a Wald Chi-square was computed from an ordinal Logistic Regression Analysis. This section also uses a *t*-statistic to compare groups on two subscales composed of multiple items. Altogether, 23 items and 2 subscales were investigated.

The results reported in this section are supplemental to the comparisons of overall scores on Test 1, Test 2, and Test 3 reported earlier in this chapter. A Bonferroni adjustment was used to keep an overall within-section alpha of .05, as insurance against putting too much weight on chance results. Since this section made 25 comparisons between the Traditional cohort and, a Bonferroni adjustment requires a significance level greater than .05/25, or approximately .002 to be relatively confident a difference was non-chance.

Evaluating expressions. Questions 3.1 and 3.2 on Test 1 required students to evaluate expressions presented in context, with access to graphing calculators. Question 7 on Test 2 required students to evaluate an expression presented without context or access to a graphing calculator. Figure 4 displays the three items.

Figure 4. Evaluating expressions.

The builders of a new American Eagle motorcycle plan to set a fixed price for the cycle—no rebates or bargaining allowed. The problem is finding the right price to charge. They estimate that their operating <i>costs</i> c and <i>sales income</i> I will be related to the number n of cycles that are made and sold: Operating costs: $c = 800n + 10,032,000$ Income from Sales: $I = 4500n - 0.2n^2$ Test 1, Question 3.1: What will the operating cost be if 5000 cycles are made and sold? Test 1, Question 3.2: What will the income from sales be if 2000 cycles are sold? Test 2, Question 7: If $s = 2$ and $t = -3$, then what is $2st + (s^2t)$?			
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Table 12 shows that students in both the Traditional and the Reform cohorts did relatively well on all three problems, with little difference between groups.

Table 1. Scores on Evaluating Expressions

	<u>Question 3.1</u>	<u>Question 3.2</u>	<u>Question 7</u>
Traditional cohort	82.9%	80.1%	79.7%
	($n=196$)	($n=196$)	($n=199$)
Reform cohorts	85.4%	79.8%	81.9%
	($n=457$)	($n=457$)	($n=443$)
χ^2 Significance Level	.320	.806	.394

Simplifying expressions. Questions 1 and 6 on Test 2 required students to simplify expressions with integer coefficients presented without context or access to a graphing calculator. Figure 5 displays the question stems for these two multiple-choice items.

Figure 5. Simplifying expressions with integer coefficients.

Test 2, Question 1: Which of the following expressions is equivalent to $(4x^3)(5y)$? Test 2, Question 6: Which of the following expressions is equivalent to $-3(5x - 6)$?

Table 2 shows that on these problems students in the Traditional cohort tended to outperform students in the Reform cohorts. The advantage of Traditional cohort students on Question 1 was statistically significant.

Table 2. Scores on Simplifying Expressions with Integer Coefficients

	<i>n</i>	Question 1	Question 6
Traditional cohort	199	59.0%	61.3%
Reform cohorts	443	50.7%	57.6%
χ^2 Significance Level		.0004	.046

Questions 2 and 4 on Test 2 required students to simplify expressions with involving ratios, presented without context or access to a graphing calculator. Figure 6 displays the question stems for these two multiple-choice items.

Figure 6. Simplifying expressions involving ratios.

Test 2, Question 2: Which of the following expressions is equivalent to $\frac{125 + x}{25}$?
Test 2, Question 4: If $t \neq 0$, which of the following expressions is equivalent to $\frac{8 - \frac{5}{t}}{2t}$?

Table 3 shows that there was essentially no difference between Traditional and Reform cohorts in students' ability to simplify expressions involving ratios. This skill was noticeably more difficult for students in both Traditional and Reform cohorts than was simplifying expressions with integer coefficients.

Table 3. Scores on Simplifying Fractional Expressions

	<i>n</i>	Question 2	Question 4
Traditional cohort	199	33.2%	24.5%
Reform cohorts	443	32.1%	28.4%
Chi-sq Significance Level		.696	.154

Solving linear equations. Three items on Test 2 required students to solve linear equations or inequalities without context or access to graphing calculators. Questions 8 and 9 presented linear equalities in an open-ended format, and Question 3 presented linear inequality in multiple-choice format. Figure 7 displays Questions 8 and 9, plus the question stem for Question 3.

Figure 7. Linear equations or inequalities without context or access to graphing calculators.

Test 2, Question 8: Solve the equation $3x + 4 = 5x - 2$ for x .	
	$-2x + 3y = 8$
Test 2, Question 9: Solve the system of equations	$x - y = 2$
Test 2, Question 3: Which of the following inequalities is equivalent to $4x - 7 > 3$?	

Table 4 shows that students in both the Traditional and the Reform cohorts did quite well on the relatively easy Question 8, with essentially no difference between them in scores. Traditional cohort students were somewhat stronger than Reform cohort students at solving the system of linear equations in Question 9, but the difference was not statistically significant. On Question 3, students in the Traditional cohort outperformed students in the Reform cohorts, and the results were statistically significant.

Table 4. Scores on Solving Linear Equations, without Context or Access to Calculators

	<u>n</u>	<u>Question 8</u>	<u>Question 9</u>	<u>Question 3</u>
Traditional cohort	199	92.1%	58.6%	47.2%
Reform cohorts	443	92.7%	52.8%	38.8%
χ^2 Significance Level		.904	.152	.002

Two items on Test 1, Questions 2.3 and 2.4, required students to translate and solve linear equations presented in context, with a graphing calculator available. Figure 8 displays the two items.

Figure 8. Linear equations in context.

The Turtle Mountain Spring Company made plans about growth in their own share of the water business. They predicted that annual income from sale of their
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bottled water B and filters F would change over time according to the following formulas. Time t is in years since 1990 and income is in millions of dollars per year.

$$\text{Bottled Water Income: } B = 20 + 5t$$

$$\text{Filtering Devices Income: } F = 28 + 3t$$

Test 1, Question 2.3: When is the income from bottled water projected to reach \$50 million?

Test 1, Question 2.4: When does the Turtle Mountain Springs Company expect the two water products to give the same annual income?

Table 5 shows that students in both the Traditional and Reform cohorts performed reasonably well on these two items. Traditional cohort students performed somewhat better than did students in the Reform cohorts on Question 2.3 but the difference was not statistically significant. In contrast, Reform cohort students outperformed Traditional cohort students on question 2.4, and the advantage approached statistical significance.

Table 5. Scores on Solving Linear Equations in Context

	<i>n</i>	Question 2.3	Question 2.4
Traditional cohort	196	71.9%	60.5%
Reform cohorts	457	66.7%	71.2%
χ^2 Significance Level		.091	.003

The possible advantage of the Reform cohorts on Question 2.4 is consistent with results reported in previous research. Huntley, et al. (2000) found that students using Core-Plus Mathematics greatly outperformed Control students on this particular question.

It is unclear why students in the Reform cohorts appear to have been more advantaged on Question 2.4 than on Question 2.3. One possible explanation is that Question 2.4 can be solved by finding the intersection of two straight lines, a problem type that may be particularly amenable to solution using the graphing calculator skills emphasized by IMP and similar curricula.

Solving quadratic equations. Three items on Test 2, Questions 10, 11, and 13, required students to solve equations presented in a standard format, without context or access to graphing calculators. Figure 9 displays the three items.

Figure 9. Quadratic equations without context or access to a graphing calculator.

Question 10: Solve the equation $x^2 + 3x - 4 = 0$ for x. Question 11: Solve $(2x - 6)(x + 4) = 0$ for x. Question 13: Solve $\sqrt{9 + x^2} = 5$ for x.
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Table 6 shows that students both in the Traditional cohort and in the Reform cohorts found these quadratic equations to be much more challenging than the linear

Table 6. Scores on Solving Quadratic Equations Presented without Context or Use of Graphing Calculators

	<i>n</i>	Question 10	Question 11	Question 13
Traditional cohort	199	39.7%	36.0%	21.8%
Reform cohorts	443	32.5%	33.1%	22.7%
χ^2 Significance Level		.143	.726	.747

equations described earlier. Students in the Traditional cohort outscored students in the Reform cohorts on the Questions 10 and 11, which were presented in standard format. Students in the Reform cohort outscored students in the Traditional cohort on Question 13, which was presented in a non-standard format. None of the differences were statistically significant.

Two items on Test 1, Questions 3.3 and 3.4, required students to solve quadratic equations in a realistic context, with a graphing calculator available. The items are displayed in Figure 10.

Figure 10. Quadratic equations in context.

The builders of a new American Eagle motorcycle plan to set a fixed price for the cycle—no rebates or bargaining allowed. The problem is finding the right price to charge. They estimate that their operating *costs* c and *sales income* I will be related to the number n of cycles that are made and sold:

$$\text{Operating costs: } c = 800n + 10,032,000$$

$$\text{Income from Sales: } I = 4500n - 0.2n^2$$

Test 1, Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

Test 1, Question 3.4: For what number of cycle sales will income be as large as possible, and what will that income be?

To answer Question 3.3 correctly, students needed to realize that the answer would not be a single number, but rather a range between the two roots of a quadratic equation. Nonetheless, they could receive partial credit by correctly identifying at least one of the roots. Rather than reporting mean scores for Question 3.3, the current study investigated the more interesting question of whether students understood that a single number was an insufficient answer. Table 18 reports the percentage of students answered the question completely (scoring 2 out of 2 points) or at least identified both roots (scoring 1.5 out of 2 points).

Question 3.4 required students to find the maximum of a quadratic function, and report both the value of this maximum (I) and the value of n at which the maximum was obtained. Students received partial credit (scoring 1 out of 2 points) if they correctly reported at least one of these two values. Rather than reporting mean scores for Question 3.4, Table 7 reports whether students were able to find the maximum, i.e., whether they scored 1 point or better.

Table 7. Percent of Students Showing a Good Understanding of Problems Involving Quadratic Equations Presented in Context, with Access to Graphing Calculators

	<i>n</i>	Percent who Identified 2 roots in Question 3.3	Percent who Identified Maximum in Question 3.4
Traditional cohort	196	7.1%%	18.4%
Reform cohorts	457	5.7%	24.5%

χ^2 Significance Level

.480

.102

Table 7 shows that only a very small percentage of students in either the Traditional or the Reform cohorts identified both roots in order to answer Question 3.3. Although a larger percentage of students found the maximum for Question 3.4 than identified both roots for Question 3.3, the numbers were still relatively small. Differences between cohorts on these two problems were not statistically significant.

Overall, the data in Table 6 and Table 7 indicate that students in both Traditional and Reform cohorts had substantial room for improvement in solving quadratic equations. However, this might be misleading. Differences in achievement between students in the Traditional cohort and students in the Reform cohorts might be due to a different mix of courses under the Reform and the Traditional program, or to different amounts learned in analogous courses under the Reform and the Traditional program. In order to assess differences due to both these sources, the Algebra Achievement test was administered to all eleventh graders, regardless of what coursework they had completed. Because of this, some students in each cohort had not completed coursework dealing with quadratic equations at the time they took the Algebra Achievement test. How did opportunity to learn affect students' ability to solve these quadratic equations? Table 8 presents achievement results for each of the five questions requiring students to solve quadratic equations, broken down by mathematics program status and by highest level algebra course completed prior to testing.

The number of students reported in Table 8 differs slightly from that in Table 6 and Table 7, because a few students whose transcripts were unavailable due to the fact that they could not be matched to a student id number were dropped from the data set.

Like Table 6 and Table 7, Table 8 reveals little difference in ability to solve quadratic equations between students in the Traditional and Reform cohorts, even when data is broken down by opportunity to learn.

Table 8. Achievement in Solving Quadratic Equations, Reported by Highest Level Algebra Course Student Enrolled in Through 11th Grade

	Mean	Mean	Mean	Pct who could i.d. 2 roots	Pct who found max
	Score on Quest. 10	Score on Quest. 11	Score on Quest. 13	Quest. 3.3	Quest. 3.4
Students who had not completed core curriculum.					
Traditional cohort	14.7%	7.8%	7.0%	0%	6.4%
	(n=43)	(n=43)	(n=43)	(n=47)	(n=47)
Reform cohorts	7.8%	9.1%	6.1%	0%	5.1%
	(n=153)	(n=153)	(n=153)	(n=158)	(n=158)

Non-honors students who had completed core curriculum (Alg 2 or Integrated Math 4)

Traditional cohort	19.2%	17.5%	9.2%	1.3%	2.6%
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	(n=80)	(n=80)	(n=80)	(n=76)	(n=76)
Reform cohorts	12.9%	11.7%	12.4%	1.4%	16.1%
	(n=140)	(n=140)	(n=140)	(n=143)	(n=143)

Honors students who had completed core curriculum (Alg 2 H or Integrated Math 4 H)

Traditional cohort	76.2%	73.3%	46.2%	19.1%	44.1%
	(n=70)	(n=70)	(n=70)	(n=68)	(n=68)
Reform cohorts	76.9%	78.5%	49.9%	15.5%	52.3%
	(n=147)	(n=147)	(n=147)	(n=155)	(n=155)

Students enrolled in programs other than Honors comprised roughly two thirds each of the Traditional and Reform cohorts. Even among Traditional cohort students who had completed Algebra 2 or Reform cohort students who had completed Integrated Math 4, only a small percentage could find the roots of a quadratic equation in either standard or non-standard format. Further, less than 10% of the non-Honors students could find two quadratic roots to solve the contextualized Question 3.3. Among non-Honors students, the only group with more than 10% finding the maximum for Question 3.4 consisted of those Reform Cohort students who had enrolled in Integrated Math 4 by the end of eleventh grade. The relatively poor performance of non-Honors students from both Traditional and Reform cohorts in solving quadratic equations, especially if the same pattern is observed at other schools, may be an area of concern for educators.

About three quarters of Honors students who had completed Algebra 2 topics (whether the IMP or the Traditional version) could find the roots of a quadratic equation presented in standard format. About half of these top students could solve a similar equation presented in non-standard format. About half of the Honors students could find the maximum value of a quadratic equation as required by problem 3.4. Less than 20% could apply their knowledge of quadratic equations to find two roots in the realistic context presented in Question 3.3.

The performance on Question 3.3 may alert educators to an area of concern. According to teachers at Suburban High School, they had not directly covered problems similar to Question 3.3. Nonetheless, it might be hoped that IMP's focus on flexible understanding of functions, combined with student practice in using graphing calculators to solve problems, might have enabled at least the Honors students to understand this

mathematics and solve this problem. As Table 19 shows, only 15% of IMP Honors students could do so, even fewer than the 19% of Algebra 2 Honors students in the Traditional cohort who could solve the problem.

Mathematics teachers at Suburban High School were able to shed some light about the reasons students performed as they did on Question 3.3. A common procedure for students using graphing calculators to solve equations of all types is to set one column of a table equal to one side of the equation, a second column of the table equal to the other side of the equation, and then to identify the position where the two columns have equal values. This procedure would lead to identification of one correct root for Question 3.3, but not a range of answers between two roots. In fact, 45.3% of all students in the Reform cohorts and 21.1% of all students in the Traditional cohort correctly identified one root. The percent reaching this particular partially correct answer was also high among honors students. It was produced by 52.6% of students in the Reform cohorts who had completed Integrated Math 4 Honors and 32.3% of students in the Traditional cohort who had completed Algebra 2 Honors.

The frequency with which students produced a point-value answer for problem 3.3, rather than a range between two values, may suggest that many were relying on the graphing calculator without paying sufficient attention to the underlying algebra needed to model the equation. It should be noted, however, that two characteristics of this particular test question may have inadvertently encouraged students to make this particular error. The first characteristic was an unfortunate aspect of the way Question 3.3 was worded. As can be seen in Figure 10, the question asked students to find “what number of cycles” solved the problem. The wording could perhaps have misled them

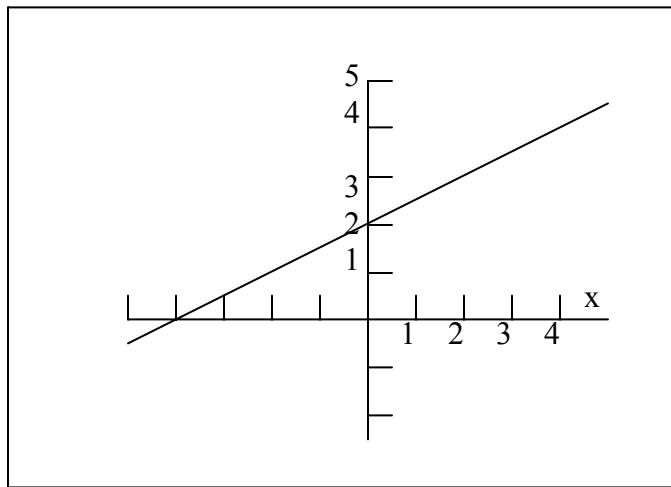
into looking for one specific number, rather than a range of values. Also, a close look at the numbers used in Question 3.3 indicates that solving the problem without use of a graphing calculator would require extensive arithmetic manipulations, accompanied by the possibility of making arithmetic errors. Thus, the arithmetic structure of the problem encouraged students to use the graphing calculator, and perhaps increased the frequency of responses based on an overly simplistic use of that particular tool.

Representational fluency: translating the graph of a line to an equation. Two of the problems on Test 2 required students to perform what traditionally is a well-practiced procedure in algebra 1: Given the graph of a line, write the equation for that line. Students were allowed to use a graphing calculator for one of these items, Question 1.2 from Part 1, Form C, but not for the other item, Question 14 from Part 2, Form A. The two items are displayed in Figure 11.

Figure 11. Translating the graph of a line into an equation.

Translating the Graph of a Line into an Equation

Question 1.2: Write an equation relating x and y that will produce this graph.



Question 14: What is an equation of the line shown in the graph below?

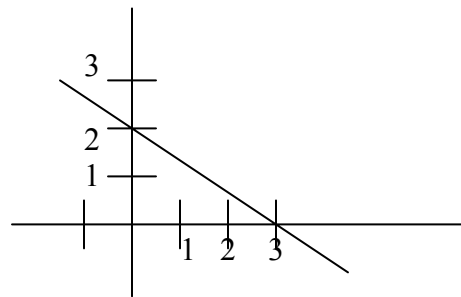


Table 9 shows that on these problems students in the Traditional cohort outperformed students in the Reform cohorts. The advantage of Traditional cohort students on Question 14 was statistically significant.

Table 9. Scores on Translating the Graph of a Line into an Equation

	Question 1.2	Question 14
Traditional cohort	63.0%	56.6%
	(n=196)	(n=199)
Reform cohorts	54.6%	39.5%
	(n=457)	(n=443)
χ^2 Significance Level	.025	<.000005

Discussions with mathematics teachers at Suburban High School about student performance on Question 1.2 and Question 14 led to a partial explanation of the Reform cohorts' weakness in this particular skill. According to the teachers, there is one module in each of the four IMP textbooks that extensively addresses the connection between linear data, linear equations, and the graph of a line. These modules are: *Overland Trail*, *Cookies*, *Small World, Isn't It?* and *The World of Functions*. However, in the *Overland Trail* and *Cookies* students most often translate from data or equations to the graph; procedures for translating from the graph of a line to its equation aren't emphasized until the module *Small World, Isn't It?* in the Year 3 textbook. *World of Functions* revisits this skill from another perspective. As reflected in Chapter 3, Table 2, during 2000-2001 Honors students did not complete *World of Functions* until they enrolled in Integrated Math 4. College Preparatory students did not complete either *Small World, Isn't It?* or

World of Functions until Integrated Math 4, and students taking Academic Assisted level courses did not complete these modules at any time.

This alone would limit opportunity to learn this concept for students in the Reform cohorts. But, in 1999-2000 the Integrated Math 4 syllabus was accommodating the fact that in previous years *Orchard Hideout* had not been completed in Integrated Math 3 College Preparatory. Therefore, in 1999-2000 *Orchard Hideout* was taught as part of Integrated Math 4 College Preparatory, and *World of Functions* was omitted.

Student achievement data lend some support to the theory that completing *Small World, Isn't It?* and/or *World of Functions* could have an effect on students' ability to translate the graph of a line to its equation. Table 21 reports the scores on Question 1.2 and Question 14 for students who at the time they were administered the Algebra Achievement test had completed neither *Small World, Isn't It?* nor *World of Functions*, for students who completed only *Small World, Isn't It?*, and students who completed both modules.

Table 21 provides some evidence that completing *Small World, Isn't It?* may have improved students' abilities to translate the graph of a line into its equation. It should be noted, however, that students who completed *Small World, Isn't It?* tended to be higher in mathematics interest and/or ability than those who did not.

Non-Honors students who completed only the module *Small World, Isn't It?* were similar in motivation and ability to non-Honors students who completed both *Small*

Table 10. Opportunity to Learn: Scores on Translating the Graph of a Line into an Equation for Reform Cohort Students who Completed Relevant Modules

	Question 1.2	Question 14
Completed neither	19.0% (n=146)	6.8% (n=143)
Completed <i>Small World, Isn't It?</i>	59.2% (n=65)	32.2% (n=61)
Completed both <i>Small World, Isn't It?</i> and <i>World of Functions</i>	61.1% (n=61)	45.0% (n=60)
Completed both <i>Small World, Isn't It?</i> and <i>World of Functions</i> (Honors Students)	86.0% (n=152)	74.1% (n=144)

World, Isn't It? and *World of Functions*. The two groups enrolled in the same courses,

but the syllabus differed, depending on when the student enrolled. Thus, Table 21 provides reasonably good evidence that completing *World of Functions* improved students' ability to solve Part 2, Question 14, which required students to translate the graph of a line into an equation without the help of a graphing calculator, but did not improve students' ability to solve Part 1, Question 1.2, which tested the same skill, but permitted use of a graphing calculator.

Nonetheless, even with full opportunity to learn, students in the Reform cohorts under-performed students in the Traditional cohort on this skill. Traditional cohort students who completed Algebra 2 Honors by the time of testing are an appropriate comparison group for Reform cohort students who completed the Honors version of both *Small World, Isn't It?* and *World of Functions*. Each comprises the top 35% of their respective cohort. On Question 1.2, Traditional Honors students scored approximately the same (86.9%) as Honors students in the Reform cohorts (86.0%), but on Question 14, Traditional Honors students scored better (89.6% vs. 74.1%).

Traditional cohort students who completed Algebra 2 by the time of testing are a fairly appropriate comparison group for students in the Reform cohorts who completed

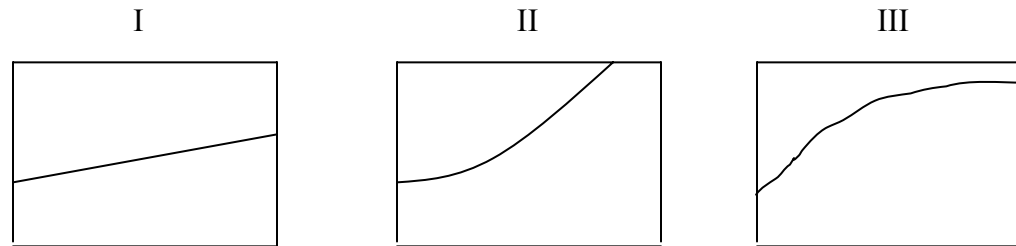
either *Small World, Isn't It?* alone or both *Small World, Isn't It?* and *World of Functions*. The Traditional students in this group comprise the middle 40% of the Traditional cohort, and the Reform students comprise the middle 29% of the Reform cohorts. On both questions requiring students to translate a linear graph into an equation, Traditional cohort Algebra 2 students outscored non-Honors students in the Reform cohorts, even when the Reform cohort students had completed both modules. On Question 1.2, Traditional cohort students who had completed algebra 2 scored 74.3%, versus 61.1 % for Reform cohort students who had completed both modules. On Question 14, Traditional cohort students who had completed algebra 2 scored 56.1%, versus 45.0% for Reform cohort students who had completed both modules.

Representational fluency: Non-procedural tasks. Two additional items on the Algebra Assessment were designed to measure students' ability to translate information presented in a graphical format: Test 1, Questions 2.1 and 2.2. These items are displayed in Figure 12.

Figure 12. Translating graphs (non-procedural).

Recent reports about bacteria and parasites in city drinking water supplies have led to a booming business in sales of bottled water and water purifying devices. Water has become a big business.

The following graphs show three different predictions of how income from water sales might change over the next several years.



Test 1, Question 2.1: Which of the three graphs (I, II, or III) shows annual income rising:

at an increasing rate? _____

at a constant rate? _____

at a decreasing rate? _____

Test 1, Question 2.2: Income from sales of bottled water in 1990 was \$500 million. What is the best match between the three graphs and these equations relating sales income S in millions of dollars to time t in years since 1990?

$S = 500 + 60t$ _____

$S = -15t^2 + 290t + 500$ _____

$S = 500 (1.2)^t$ _____

Table 22 shows that on these less procedural translation problems students in the Reform cohorts outperformed students in the Traditional cohorts. The advantage of Reform cohort students on Question 2.2 was statistically significant.

Table 11. Scores on Interpreting and Translating Line Graphs

	<i>n</i>	Question 2.1	Question 2.2
Traditional cohort	191	89.5%	59.7%
Reform cohorts	456	94.7%	73.8%
χ^2 Significance Level		.010	.00004

Question 1.4 on Test 1 required students to identify an exponential function presented in tabular form, and translate it into an equation. Figure 13 displays the question.

Figure 13. Exponential function in tabular form.

Test 1, Question 1.4: Write an equation relating x and y that will give the pairs of numbers in this table.					
x	-1	0	1	2	3
y	0.5	1	2	4	8

Table 12 shows that while this item was difficult for students in all cohorts, students in the Reform cohorts were about twice as likely to get the item correct as were students in the Traditional cohort. The advantage of Reform cohort students on Question 1.4 was statistically significant.

Table 12. Scores on Translating an Exponential Function

	<u>n</u>	<u>Question 1.4</u>
Traditional cohort	191	14.7%
Reform cohorts	456	28.3%
χ^2 Significance Level		.0001

Although students work with exponential functions in many IMP modules, the topic receives particular emphasis in three modules: *All About Alice*, *Small World, Isn't It*, *World of Functions*. Table 24 shows that students in the Reform cohorts who had completed a larger number of these modules were particularly advantaged in solving Question 1.4. The importance of opportunity to learn in determining student achievement is emphasized by the fact that students who completed only the modules *All About Alice* and *Small World Isn't It?* were those students enrolled in Integrated Math 4 College Preparatory before the 2000-01 school year, while those non-Honors students who completed the same two modules plus *World of Functions* were those students enrolled in the same Integrated Math 4 College Preparatory class during the 2000-01 school year, when *World of Functions* was added to the course syllabus. Although students in the two groups were of comparable ability and motivation, Table 24 shows that students who completed *World of Functions* scored much higher on Question 1.4 than did students who did not.

Table 13. *Ability to Translate an Exponential Function to an Equation*

	<u><i>n</i></u>	Mean Score on Quest 1.4
Traditional Cohort		
<u>Highest Enrolled Course</u>		
< Algebra 2	47	2.1%
Algebra 2	76	2.6%
Algebra 2 Honors	68	36.8%
Reform Cohorts		
<u>Relevant Modules Completed</u>		
none	57	3.5%
<i>All About Alice</i>	105	4.3%
<i>All About Alice</i> and <i>Small World, Isn't It?</i>	70	11.4%
<i>All About Alice</i> and <i>Small World, Isn't It?</i> and <i>World of Functions</i>	69	31.9%
<i>All About Alice</i> and <i>Small World, Isn't It?</i> and <i>World of Functions</i> (Honors Students)	155	59.7%

Formulation and interpretation of mathematical models. As described by

Huntley, et al. (2000), many items on Test 1 asked students either to formulate an algebraic model or to interpret results from use of a given algebraic model. The present study used a total of 10 items that fit into this category.

Five of these items were combined to create a “Mathematical Formulation”

subscale: Questions 1.1, 1.3, 1.5a, and 1.5b from Part 1, Form C of the Algebra

Achievement test, and Question 5 from Part 2, Form A. The Mathematical Formulation subscale had a reliability (Cronbach’s alpha) of 70.4%.

Figure 14 displays Question 1.1. The remaining items are displayed in Appendix A.

Figure 14. Formulating an algebraic model.

Part 1, Question 1.1: The Watchdog Security Service provides home and business security systems with installation charge of \$150 and a charge of \$5 per week to monitor the system. Write a formula that gives the cost C of the service as a function of time t in weeks that the service is needed.

Table 14 reports differences between the Traditional and Reform groups on percent correct of the Mathematical Formulation subscale. As the table indicates, students in the Reform cohorts averaged 58.0% correct on the Mathematical Formulation subscale, significantly better than the 49.3% achieved by students in the Traditional cohort ($p<.0003$).

Table 14. Difference Between Groups on Mathematical Formulation

Treatment		<i>n</i>	Percent Correct		Standard Deviation	
Reform		419	58.0%		28.5%	
Traditional		191	49.3%		26.3%	

Difference of Scores	<i>t</i>	df	<i>p</i> -value	95% Conf. Interval		99.8% Conf. Interval	
				Lower Bound	Upper Bound	Lower Bound	Upper Bound
8.8%	3.61	608	.0003	4.0%	13.5%	1.2%	16.3%

The standard deviation for Mathematical Formulation scores in the Traditional cohort was 26.3%. Reform group students scored 8.8% better than did Traditional group students. This is an effect size of 8.8/26.3, or approximately .33 standard deviations. Assuming that student abilities are normally distributed, then if a student would ordinarily score at the 50th percentile, improving his or her score by .33 standard deviations would move the student up to the 63rd percentile.

Each of the five questions comprising Problem 4 on Test 1 called for students to interpret the meaning of algebraic calculations. For the present study, these five questions were combined into a single “Interpreting Algebraic Models” subscale. The Interpreting Algebraic Models subscale had an estimated reliability (Cronbach’s alpha) of 88.7%. Note, however, that because the five items making up the subscale are all part of the same super-problem, it is possible that this reliability estimate is somewhat inflated due to non-independence among student responses.

Figure 15 displays the first three questions in Problem 4. The remaining two questions are displayed in Appendix A.

Figure 15. Interpreting an algebraic model.

Several commercial airlines have non-stop flights from Los Angeles, California all the way to Sydney, Australia. It is a trip of 7500 miles and can take as long as 18 hours, most of the time out over the Pacific Ocean. Therefore, estimating flight time and fuel requirements is very important. One airline uses this formula to calculate flight time T in hours from expected wind speed W in miles per hour:

$$T = \frac{7500}{500 + W}$$

Test 1, Question 4.1: What will you learn from calculating T when $W=75$?

Test 1, Question 4.2: What will you learn from solving: $\frac{7500}{500 + W} = 14$?

Test 1, Question 4.3: What will you learn from solving: $\frac{7500}{500 + W} > 14$?

Table 15 reports differences between the Traditional and Reform groups on percent correct of the Interpreting Algebraic Models subscale. As the table indicates, students in the Reform cohorts averaged 49.9% correct, compared to 44.5% for students in the Traditional cohort. The advantage of Reform cohort students was not statistically significant ($p<.065$).

Table 15. Difference Between Groups on Interpreting Algebraic Models

Treatment		<i>n</i>	Percent Correct		Standard Deviation	
Reform		457	49.9%		35.0%	
Traditional		196	44.5%		32.7%	

				95% Conf. Interval		99.8% Conf. Interval		
Difference of Scores		<i>t</i>	df	<i>p</i> -value	Lower Bound	Upper Bound	Lower Bound	Upper Bound
5.4%		1.85	651	.065	-2.9%	+11.2%	-3.7%	+14.3%

The standard deviation for Interpreting Algebraic Models scores in the Traditional cohort was 32.7%. Reform group students scored 5.4% better than did Traditional group students. This is an effect size of $5.4/32.7$, or approximately .17 standard deviations.

Assuming that student abilities are normally distributed, then if a student would ordinarily score at the 50th percentile, improving his or her score by .17 standard deviations would move the student up to the 57th percentile.

Summary of detailed differences between groups. Underlying the difference between the Reform cohorts and the Traditional cohort on Test 1 and Test 2 were apparent patterns of differences in specific sub-skills. Although the advantage of students in the Traditional cohort did not pass the Bonferroni-adjusted statistical significance test used in this study, they were observed to outscore Reform students on certain sub-skills within Test 2. When there was a difference between groups on questions involving specific, well-practiced procedures, students in the Traditional cohort tended to do better. Students in the Traditional cohort outperformed students in the Reform cohorts at the specific skill of writing an equation for a line, when presented a graph of that line—especially when this skill was performed without access to a graphing calculator. They also outscored the Reform cohorts on simplifying expressions, and on solving linear and quadratic equations when presented symbolically in standard form. However, they had no advantage in solving expressions involving ratios, or in solving quadratic equations when presented symbolically in a non-standard form, or in solving linear or quadratic equations when presented as part of a realistic problem context. Underlying their significantly higher achievement on Test 1, students in the Reform cohorts were noticeably better than students in the Traditional cohort at several particular sub-skills. They were consistently better at formulating a mathematical model to describe a problem situation. They also tended to be better at interpreting algebraic models, although the results were statistically significant. Finally, they were better at all three non-procedural items that measured their ability to translate among differing representations of functions. They were better at interpreting graphs on a Cartesian plane, and at estimating what type of equation might fit the graph. They were also better at recognizing an exponential function presented in tabular form and writing an equation for that function.

Effects of Measuring Achievement the end of Grade 11

Instead of at Other Possible Measurement Times

To put the results of the Algebra Achievement test in context, two additional analyses were undertaken investigating how the timing of the assessment might have affected those results. The first analysis compared scores on Test 1 and Test 2 of

students in Reform cohorts who were enrolled in a mathematics course at the time of testing to scores on Test 1 and Test 2 of students in Reform cohorts who were not enrolled in any mathematics course during the semester in which they completed the Algebra Achievement test. The second analysis used student transcripts to see whether the Algebra Achievement test was likely to have captured most of the algebra learning students completed in high school, or whether in contrast students tended to study a significant amount of algebra in twelfth grade, after they had completed the test.

Possible underestimate of algebra achievement for those students not currently

enrolled in mathematics. The current study measured student algebra achievement at the

end of eleventh grade. The soonest students were likely to apply this knowledge was at the beginning of twelfth grade, at the end of a 3-month summer recess. For this reason, it is possible that a delayed test, administered at the very beginning of twelfth grade, would have been a better way to measure student algebra achievement. That way, all students would have experienced a summer recess during which they may have forgotten some of the mathematics they had learned. If much of what is forgotten is forgotten quickly, then those students in the Reform cohorts who were not enrolled in mathematics during the spring semester would likely already have forgotten much of what they were going to forget by the end of eleventh grade. Thus, students who had been enrolled in spring courses would have forgotten relatively more over the summer.

Data available from the current study provided one way to check whether the test was likely to have been biased against students not currently enrolled in mathematics.

After controlling for prior ability, the achievement of Reform Cohort students enrolled in mathematics class at the time of testing was compared to that of Reform Cohort students who were not enrolled in mathematics at the time of testing on Test 1 (Understanding and Solving Algebra Problems presented in Context and Test 2 (Algebra Symbol Manipulation). The comparison indicated that on Test 2 there may have been such a bias.

On Test 1, Reform cohort students who were enrolled in mathematics class at the time of testing scored similarly to those not enrolled in mathematics class at the time of testing: 58.2% correct for those currently enrolled, compared to 58.3% for those not currently enrolled. On Test 2, Reform cohort students who were enrolled in mathematics class at the time of testing scored slightly higher than those not enrolled in mathematics class at the time of testing: 54.2% correct for those currently enrolled, compared to

52.0% for those not currently enrolled. However, for reasons not explained by data available in this study, the students enrolled in mathematics during the second semester of eleventh grade tended to have lower sixth-grade ability scores than students who were not enrolled in mathematics during the second semester of eleventh grade. Therefore, after controlling for prior ability, Reform cohort students currently enrolled in mathematics class scored significantly higher on Test 2 than did Reform cohort students not currently enrolled in mathematics class ($p = .024$). On Test 1, after controlling for prior ability Reform students enrolled in mathematics class scored higher than did Reform students not currently enrolled in mathematics class, but the result was not statistically significant ($p = .335$).

A total of 28.5% of students in the Reform cohorts who completed the Algebra Achievement test were not enrolled in mathematics class at the time of testing. As administered, the test may have underestimated the achievement of these students, as compared to the achievement of other students completing the test.

Unmeasured effects of twelfth-grade coursework. Students at Suburban High

School completed the Algebra Achievement test for this study at the end of Grade 11. Gathering student achievement data from the end of Grade 12 might have provided a better measure of the complete effects of the changes to the schedule and curriculum at Suburban High School. Unfortunately, it was not believed possible to obtain a valid measure of achievement at the end of Grade 12. The Algebra Achievement test was intentionally administered in such a way as to keep it a low stakes test in order to avoid creating any reason for faculty to contaminate the results by teaching to the test. Most student tests were not even scored nor results reported until several years after they were administered. However, while it seemed reasonable to expect valid results from a low-stakes test administered to students near the end of eleventh grade, it was deemed unlikely that participation and effort would be high quality if the test were administered near the end of twelfth grade, just before high school graduation. It was anticipated that this would be particularly problematic under the block schedule, since a large number of twelfth graders would not even be enrolled in mathematics class at the time the test was administered.

Student transcripts were analyzed to determine what percent of students were likely to have studied a significant amount of algebra in twelfth grade, subsequent to administration of the Algebra Achievement test. Because students in the Traditional cohort who were tested in the spring of 1997 did not utilize a traditional schedule in twelfth grade, their transcripts were not used in this analysis. Instead, transcripts from

the two preceding cohorts, who graduated, respectively, in the spring of 1997 and the spring of 1998, were analyzed. Also, to analyze twelfth grade algebra course-taking for students using the semestered block schedule and IMP curriculum only transcripts from the First Reform cohort were utilized, because students in the Second Reform cohort had not completed twelfth grade at the time this investigation was conducted.

Under the traditional schedule and curriculum, some students studied Algebra 2 for the first time in twelfth grade, and others revisited the topic in a course called Algebra 3/Trigonometry that utilized the UCSMP text *Advanced Algebra* (Senk & Thompson, 1993). *Advanced Algebra* uses an alternative approach to traditional texts that past research has shown to be very successful (Thompson & Senk, 2001). During the 1995-96 school year, 36.4% of all twelfth graders studied algebra in one of the other of these two ways. During the 1996-97 school year, 34.3% of all twelfth graders studied algebra in one of the other of these two ways.

Of all students in the First Reform cohort who completed the Algebra Achievement test in eleventh grade, 32.5% subsequently enrolled in an Integrated Mathematics class during twelfth grade. Of the 27 students in the First Reform cohort who were administered the Algebra Achievement test and who eventually completed Integrated Math 4 Academic Assisted, 22 (81.5%) did so in twelfth grade. Of the 92 students in the First Reform cohort who were administered the Algebra Achievement test and who eventually completed Integrated Math 4 Academic Assisted, 41 (44.6%) did so in twelfth grade.

In summary, under both the Traditional and the Reform program, about one-third of all students at Suburban High School studied some form of algebra course during twelfth grade. The current study did not capture the effects of whatever algebra they learned at that time. It is particularly regrettable that most of the students in the Reform cohorts who were in Academic Assisted classes had not yet completed the core IMP sequence at the time of testing. Under the new program, Academic Assisted students were offered exposure to more sophisticated mathematics than had previously been the case, and it would have been desirable to measure the full effects of the program on this particular group.

Transcript Analysis

This study examined an automated database containing complete transcripts of Suburban High School students beginning with class who graduated in June, 1996 as well as partial transcripts from earlier cohorts of students. The transcripts were used for two purposes.

First, the study analyzed the total number of hours during which students were enrolled in mathematics classes and in advanced mathematics classes from the beginning of Grade 9 through the end of Grade 12. Students who used a traditional curriculum and a traditional schedule for all four years of high school were compared to students who had used the IMP curriculum and a semestered block schedule throughout all four years of high school.

Second, the study investigated the number of students by grade level who were enrolled in Advanced Placement courses each school year from 1994-95 through 2000-01. This analysis was supplemented with data from annual School Profiles published by Suburban High School, which reported student grades on Advanced Placement tests administered by the college board.

Results from Analysis of Complete High School Transcripts

The specific groups of students compared in the analysis of complete high school transcripts were different from the groups used to analyze algebra achievement at the end of Grade 11. Of the students in the two Reform cohorts, only students in the First Reform cohort had completed twelfth grade at the time this part of the research was conducted, so only their transcripts were used. Also, students in the Traditional cohort who were tested at the end of eleventh grade in May 1997 went on to use a semestered block schedule during the 1997-98 school year. For this reason, their transcripts were not used for the analysis of transcripts through the end of Grade twelve. Instead, this part of the study analyzed transcripts of students from the two previous “Traditional” cohorts: those students who graduated in the spring of 1997, and those who graduated in the spring of 1996. Students who graduated in those years used a traditional curriculum and schedule throughout high school.

Students who attended Suburban High School from the beginning of Grade 9 through the end of Grade 12 and graduated in the spring of 2001 comprise the Reform group ($n = 234$) for this part of the analysis. Students who attended Suburban High School from the beginning of Grade 9 through the end of Grade 12 and graduated in the

spring of 1996 or the spring of 1997 comprise the Traditional group ($n = 416$) for this part of the analysis.

For students in the Traditional group, each mathematics class lasted 43 minutes per day and met daily for the entire 180-day school year. Thus, each course accounted for $43 \times 180/60 = 129$ enrolled hours.

For students in the Reform group, each mathematics class taken during ninth grade lasted 80 minutes per day and met daily for an 80 day semester. Thus, each ninth-grade course accounted for $80 \times 80/60 = 106.67$ enrolled hours. For these students, each mathematics class taken during tenth through twelfth grade lasted 80 minutes per day and met daily for a 90 day semester. Thus course in Grades ten through twelve accounted for $90 \times 80/60 = 120$ enrolled hours.

Comparison of total hours enrolled in mathematics class. Table 16 compares the number of hours each group, Traditional and Reform, enrolled in mathematics class during Grades 9-12. As the Table shows, students in the Reform Group enrolled in more hours of mathematics than did students in the Traditional Group.

Table 16. Average Number of Hours Each Student Was Enrolled in Mathematics Class, Traditional vs. Reform Groups

	<i>n</i>	Total Hours Enrolled In Mathematics, Grades 9-12 (Mean per Student)	Average Yearly Enrolled Hours (Mean per Student)
Reform Group	234	564.7	141.2
Traditional Group	416	498.0	124.5

It should be noted when students who comprised the Reform Group in Table 16 were in ninth grade, Suburban High School used the pilot form of the semestered block

schedule, under which each course lasted only 106.67 hours. Subsequent cohorts have used the modified block schedule throughout Grades 9-12, under which each course lasts 120 hours. Thus, it is likely that later Reform cohorts will report an even larger number of hours enrolled in mathematics class under the semestered block schedule than is reflected in the table.

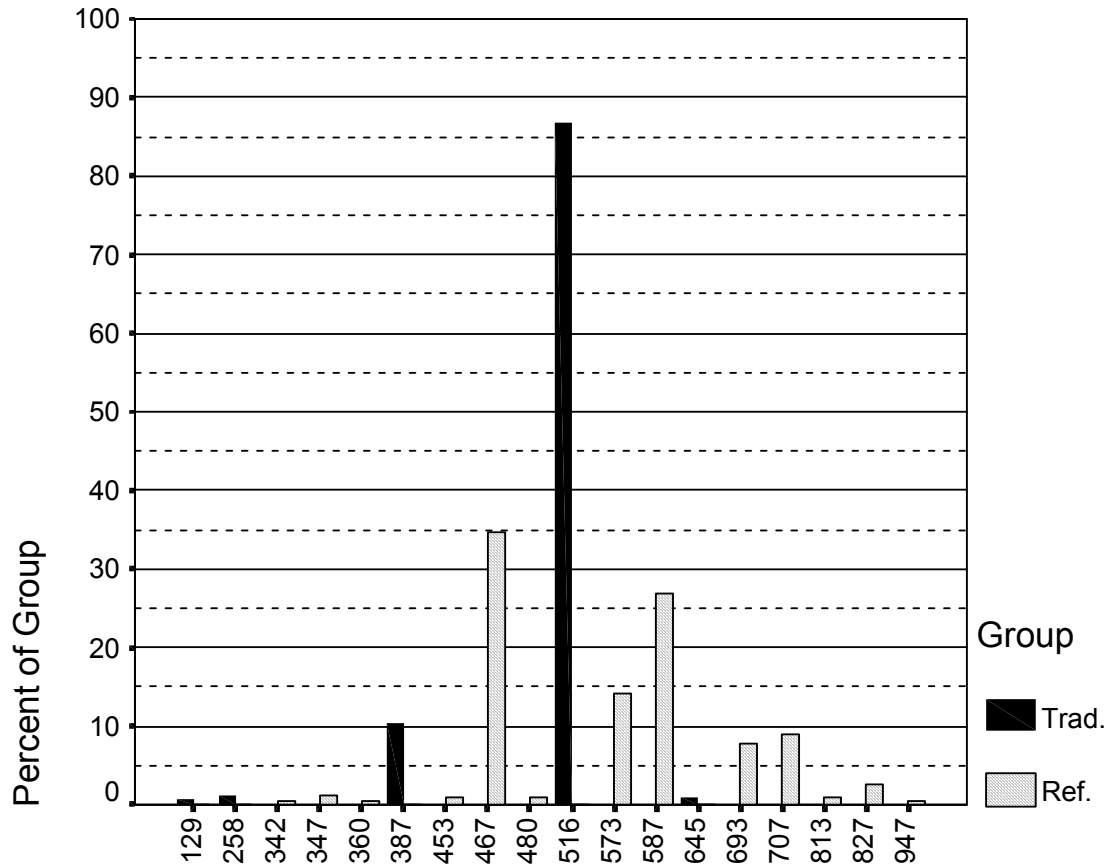
The summary statistics in Table 16 obscure some important details about the number of hours students in the Traditional and Reform groups enrolled in mathematics class. Table 17 displays more details about the total hours students in each group were enrolled in mathematics from the beginning of Grade 9 through the end of Grade 12. The Table reports the percent of students within each group who enrolled in a particular number of hours of mathematics. Mathematics courses from which students withdrew before receiving a final grade, or for which students received a final grade of “F”, did not count towards their total hours. The information in Table 17 is displayed graphically in Figure 16.

Table 17. Total Hours Enrolled in High School Mathematics for Traditional vs. Reform Groups

Hours		Traditional	Reform
129	# students	3 students	
	% within Group	0.7%	
258	# students	5 students	
	% within Group	1.2%	
342.34	# students		1 student
	% within Group		0.4%
346.67	# students		3 students
	% within Group		1.3%
360	# students		1 student
	% within Group		0.4%
387	# students	43 students	
	% within Group	10.3%	

453.34	# students	2 students
	% within Group	0.9%
466.67	# students	81 students
	% within Group	34.6%
480	# students	2 students
	% within Group	0.9%
516	# students	361 students
	% within Group	86.8%
573.34	# students	33 students
	% within Group	14.1%
586.67	# students	63 students
	% within Group	26.9%
645	# students	4 students
	% within Group	1.0%
693.34	# students	18 students
	% within Group	7.7%
706.67	# students	21 students
	% within Group	9.0%
813.34	# students	2 students
	% within Group	0.9%
826.67	# students	6 students
	% within Group	2.6%
946.67	# students	1 student
	% within Group	0.4%

Figure 16. Total Hours Enrolled in Mathematics Class



Several interesting trends are apparent in Figure 16. A large majority of students in the Traditional group enrolled in 516 hours of mathematics during high school. This is equivalent to four 129-hour courses or one mathematics course each year of high school. This was one course more than the Suburban High School graduation requirement under the Traditional schedule, which set a 3-course minimum for mathematics.

Some students in the Reform group enrolled in fewer hours of mathematics than the modal 516 hours for the Traditional group. However, Figure 16 shows that the

majority of students in the Reform group enrolled in more than 516 hours, and some substantially more. More than 20% of students in the Reform group enrolled in at least 693 hours of mathematics. In contrast, the largest number of enrolled hours for any student in the Traditional group is 645, equivalent to five traditional courses. Further, only 1% of students in the Traditional group enrolled in that much mathematics.

Under the semestered block schedule, Suburban High School required students to complete at least four mathematics courses in order to graduate. In Figure 16, students who enrolled in precisely this minimum of four courses are represented by the bars at 453, 467, or 480 hours of mathematics—the particular bar depending on what particular years they studied mathematics courses. Figure 16 shows that most students in the Reform group enrolled in more than the minimum number of mathematics courses required for graduation, as did students in the Traditional group.

Overall, 88% of students in the Traditional group enrolled in more than the minimum required number of courses, taking four or more mathematics courses during high school. Meanwhile, 62% of students in the Reform group enrolled in more than the minimum required number of courses, taking five or more mathematics courses during high school.

One way of looking at the information in Table 17 and Figure 16 is to ask what percent of students in the Reform group enrolled in more hours of mathematics than they likely would have under a Traditional schedule and curriculum. Of all students in the Reform cohort, 38% enrolled in fewer than 516 hours, the equivalent of taking four courses under the Traditional schedule. Of all students in the Traditional group, 12% enrolled in fewer than 516 hours. Thus a rough estimate is that about one quarter (38%

minus 12%) of students in the Reform group who were likely to have enrolled in 516 hours of mathematics under the Traditional schedule instead enrolled in fewer than 516 hours. The remaining three quarters of students in the Reform cohort enrolled in more hours than they would have done under the Traditional schedule. Some students in this three quarters (12% of all Reform group students) are those who would probably have enrolled in three or fewer mathematics courses under the Traditional schedule, and instead enrolled in four mathematics courses under the semestered block schedule. The rest of the students comprising this three quarters (62% of all Reform group students) are those who enrolled in five or more mathematics courses under the semestered block schedule.

Comparison of hours enrolled in advanced mathematics classes. After completing Algebra 2, students in the Traditional group could enroll in any one of the following more advanced classes: Algebraic Analysis, Algebraic Analysis Honors, Pre-Calculus/Discrete Honors, Calculus AB AP, or Calculus BC AP.

The Algebraic Analysis courses used the text *Functions, Statistics, and Trigonometry with Computers* (Rubenstein, et al., 1992). This is the fifth of six mathematics texts in a Grade 7-12 series designed by the University of Chicago School Mathematics Project (UCSMP). In sequence, it was designed to be used immediately after students had completed the Algebra, Geometry, and Advanced Algebra courses. The authors recommend that all college-intending students study material covered in the text to prepare them for the wide range of mathematics found in virtually all college majors today. The Pre-Calculus/Discrete Honors course utilized the final book in the UCSMP series, *Precalculus and Discrete Mathematics* (Perissini, et al., 1992). The text contains material intended to prepare students for Calculus and other college courses usually taken by mathematical and physical science majors. The two calculus courses

used the text *Elements of Calculus and Analytic Geometry* (Thomas & Finney, 1989).

One course offered to students in the Traditional group, Algebra 3/Trigonometry, was intended for students who had difficulty with Algebra 2, or students who completed the less advanced course Algebra 2 Career/College Prep instead of the regular Algebra 2 course. It covered Algebra 2 from a different perspective, using the text *Advanced Algebra* (Senk & Thompson, 1993). *Advanced Algebra* is the fourth book in the UCSMP series, and is recommended for all high school students, whether or not they are college-intending. Because of its content, this study did not count “Algebra 3/Trigonometry” as an “advanced class,” despite the course title.

After completing Integrated Math 4, students in the Reform group could enroll in any of the advanced courses that had been offered to the Traditional group except for the Honors version of Algebraic Analysis, which was dropped from the curriculum. Although most of the courses were renamed at about the time Suburban High School adopted the semestered block schedule, most continued to use the same texts and to present basically the same material. The two most significant changes made were to Pre-Calculus/Discrete Honors and to Calculus. Pre-Calculus/Discrete Honors was renamed Functional Analysis Honors and dropped a number of topics in discrete mathematics that were taught in a new Discrete Analysis course. The two calculus courses utilized a more recent book than did students in the Traditional cohort: *Calculus: Graphical, numerical, and Algebraic* (Finney, Demana, Waits, & Kennedy, 1999). The first author of the text, Ross Finney, had been second author of the Calculus text utilized by the Traditional cohort.

Under the semestered block schedule, four new advanced mathematics courses

were added to the Suburban High School curriculum. The new courses were: Discrete Analysis, Discrete Analysis Honors, Statistics, and Statistics AP. Both Discrete Analysis and Discrete Analysis Honors utilized the text *For All Practical Purposes* (Garfunkel, 1997) to study topics like networks, task scheduling, and weighted voting systems. Although Statistics and Statistics AP covered similar topics and used the same supplementary materials, the primary texts were different. “Statistics” utilized the text *Elementary Statistics* (Bluman, 1992), which emphasizes a non-theoretical approach, explaining concepts intuitively, supporting them by examples, and avoiding formal proofs. In contrast, “Statistics AP” utilized the text *The Basic Practice of Statistics* (Moore, 1999) which was written to be used in an introductory level college statistics course.

While Suburban High School introduced four new mathematics courses simultaneous with adoption of a semestered block schedule, it cannot be certain that the schedule alone accounted for this decision. In particular, the College Board first administered an Advanced Placement statistics examination in May of 1997. The decision to introduce an AP Statistics course at Suburban High School during the 1997-98 school year may have been as much a reaction to the newly available examination and related materials, as it was a reaction to the increased number of courses available to Suburban High School students under the block schedule.

Table 18 compares the mean number of hours students in each group, Traditional and Reform, enrolled in advanced mathematics courses during Grades 9-12. As the table shows, on average students in the Reform Group enrolled in more hours of advanced

Table 18. Difference Between Groups on Mean Hours Enrolled in Advanced Mathematics Classes

Treatment	All Students			Students With At Least One Advanced Course			
	n	Mean	Standard	n	Mean	Standard	Hours
		Deviation	Hours		Deviation	Hours	
Reform	234	132.8	143.3	135	230.2	114.5	
Traditional	416	96.8	65.4	110.8	199	202.3	

mathematics than did students in the Traditional Group. This is true among all students in the data set, and even among the set comprised of only those students who took at least one advanced mathematics course.

The summary statistics in Table 18 obscure some important details about how many hours students in the Traditional and Reform groups enrolled in advanced mathematics courses. Table 19 displays more details about the total hours students in each group were enrolled in advanced mathematics courses from the beginning of Grade 9 through the end of Grade 12. The table reports the percent of students within each group who enrolled in a particular number of hours of advanced courses. Advanced mathematics courses from which students withdrew before receiving a final grade, or for which students received a final grade of “F”, did not count towards their total hours.

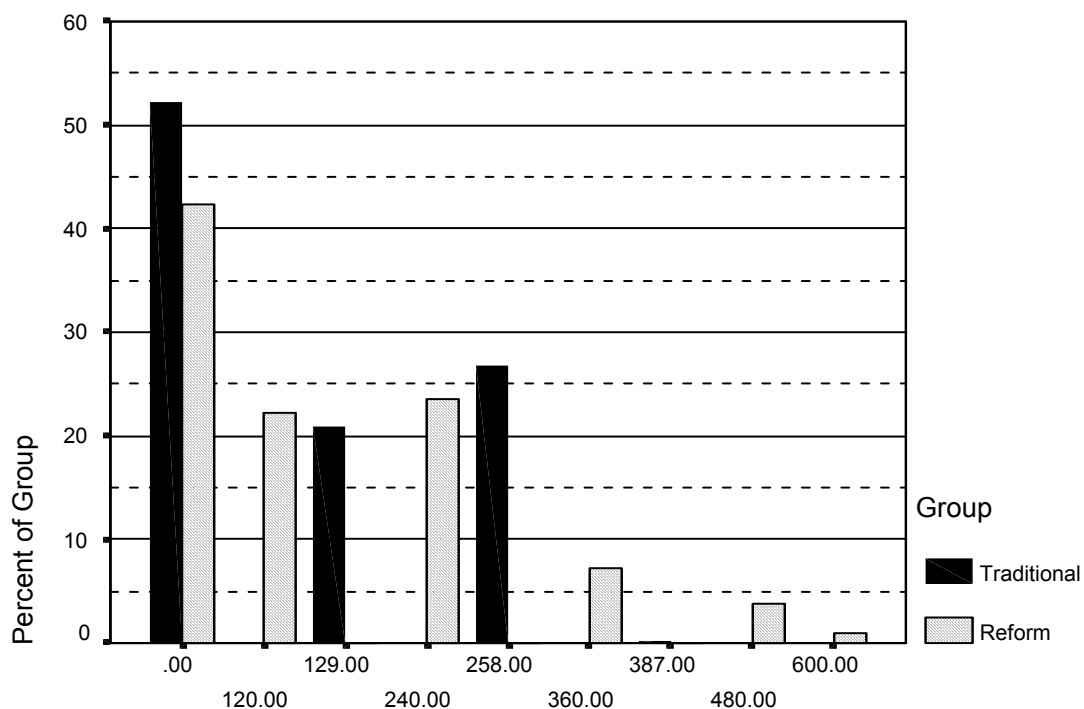
Table 19. Total Hours Enrolled in Advanced Mathematics Courses for Traditional vs. Reform Groups

Hours		Traditional	Reform
0	# students	217 students	99 students
	% within Group	52.2%	42.3%
120	# students		52 students
	% within Group		22.2%
129	# students	87 students	
	% within Group	20.9%	
240	# students		55 students
	% within Group		23.5%
258	# students	111 students	
	% within Group	26.7%	
360	# students		17 students
	% within Group		7.3%
387	# students	1 student	
	% within Group	0.2%	
480	# students		9 students
	% within Group		3.8%
600	# students		2 students
	% within Group		0.9%

The information in Table 19 is displayed graphically in Figure 17. As can be seen in Figure 17, students in the Reform group were more likely than students in the Traditional group to take at least one advanced mathematics course. In the Reform group, 57.7% of all students took at least one advanced course, compared to 47.8% of students in the Traditional group. A Pearson Chi-square statistic was computed to test the statistical significance of the difference between cohorts in the odds of a student enrolling in at least one advanced mathematics course. The test yielded a Chi-square statistic of 5.82, $p < .016$. The Chi-square test was conducted after visual inspection of the graph to look for important patterns, so the low p -value could be spurious. Nonetheless, the significance level is an indication that the difference cannot easily be dismissed as

chance.

Figure 17. Total hours enrolled in advanced mathematics courses.



For students who enrolled in one or two mathematics courses, the amount of course time scheduled was greater in the Traditional group than in the Reform group: an advantage of nine hours course-time per course. However, this deficit was balanced out by the much greater likelihood that students in the Reform cohort would take three or more advanced courses.

Overall, Figure 17 along with Table 18 and Table 19 show that students in the Reform group were more likely than students in the Traditional group to enroll in at least one advanced mathematics course. Further, 12% of students in the Reform group enrolled in three or more advanced mathematics courses, as opposed to less than 1% of students in the Traditional group who did so. Although each mathematics course

comprised fewer hours for the Reform group, the tendency of students in the Reform group to enroll in a larger number of courses more than balanced this out. Thus, on average students in the Reform group spent more hours in advanced classes than did students in the Traditional group.

In computing the number of hours students were enrolled in advanced courses, Table 18, Table 19, and Figure 17 did not count Integrated Math 4 Honors as advanced. There were several reasons for this decision. At Suburban High School all versions of Integrated Math 4 tended to be viewed as part of the core mathematics requirements. For example, before students could enroll in Discrete Analysis, Algebraic Analysis, or Statistics they were expected to complete either Integrated Math 4 or Integrated Math 4 Honors. Also, students generally completed the Honors sequence relatively early in their high school careers. Only 2 of the 84 students in the First Reform cohort who took the course did so during the 2000-2001 school year, which was the first year the Integrated Math Honors course syllabus called for all the modules in the IMP textbooks to be completed. Previously, teachers had still been familiarizing themselves with the curriculum, and therefore the Honors courses did not complete the modules *Is There Really a Difference*, *Pennant Fever*, or *The Pollster's Dilemma*. Thus, for students in the First Reform Cohort whose transcripts were analyzed in this study, Integrated Math 4 Honors probably should not count as an “advanced” course.

Nonetheless, in looking at the long term effects of the IMP curriculum and semestered block schedule, the reader should note that for subsequent cohorts Integrated Math 4 Honors probably would count as an “advanced” course. Beginning in 2000-2001, Integrated Math 4 Honors syllabus contained all modules in the entire IMP Year 4

textbook. According to teachers at Suburban High School, in 2000-2001 they did not finish the entire syllabus, leaving out the module *Is There Really a Difference?* but they anticipate completing the entire Book 4 during 2001-2002. Webb (in press) counted a course designed from a preliminary version of the IMP Year 4 text as “advanced.” Although some of the topics contained in IMP Year 4 would likely be found in a traditional Algebra 2 course, the bulk of the topics are more likely to be seen in subsequent courses. Further, students who completed the entire Integrated Math Honors sequence would have completed two modules containing advanced topics in matrix algebra and probability and statistics when they took Integrated Math 3 Honors. A graphical analysis not shown here indicated that the primary effect of counting Integrated Math 4 Honors as an “advanced course” would be a substantial increase in the reported percent of students who enrolled in 360 hours or more of advanced mathematics.

Advanced Placement Courses

As described by the College Board, their Advanced Placement (AP) program gives students the opportunity to take college-level courses and exams while they are still in high school (College Board, 2001a). The courses are intended to help students develop skills and study habits that will be vital in college. Calculus AB covers differential and integral calculus topics that are typically included in an introductory Calculus 1 college course. Calculus BC covers the Calculus AB topics as well as additional topics in differential and integral calculus, and series. The Calculus BC topics are typically included in a two-semester sequence (Calculus 1 and 2) at the college level. AP Statistics covers topics typically included in a one-semester introductory non-calculus-based college course in statistics.

AP Course Enrollment

Table 20 reports the number of students by grade level who completed an AP mathematics course each school year, from 1990-91 through 2000-01. Only students who received a passing grade are counted in the Table.

In 1997-98 Suburban High School adopted a semestered block schedule at all grade levels. In that year an elective one-semester AP Statistics course was added to the curriculum. Also, Calculus AB and Calculus BC had previously been offered as separate year-long courses under a traditional schedule. The two year-long calculus courses were replaced by a pair of sequential one-semester courses. Thus, students who wished to enroll in Calculus BC did so after completing a one-semester Calculus AB course. In Table 31, students who completed only the one-semester Calculus AB course are counted as having completed “Calculus AB”, while to avoid double counting students who completed Calculus AB followed by Calculus BC are counted as having completed “Calculus BC” but not “Calculus AB”. Between 1997-98 and 2000-01 a total of 38 students completed both AP calculus and AP statistics, frequently enrolling in the statistics course the year after completing calculus. For these students, both their AP statistics course and their highest level of AP calculus course are reflected in the Table.

Usually, students who enrolled in an AP mathematics courses were in eleventh or twelfth grade. In 1997-98 and 1998-99 eleventh and twelfth graders at Suburban High School had experienced a hybrid schedule: traditional in their early years of high school, and a semestered block schedule nearer the end of their high school career. In Table 20, entries for the two “hybrid-schedule” years are italicized. In the hybrid-schedule years, the total number of students enrolling in AP classes increased over what had been the

Table 20. Advanced Placement Mathematics Course Enrollment by School Year and Grade Level

School Year			Grade Level			Total
			10	11	12	
1990-91	Course	Calculus AB AP	-	-	26	26
		Calculus BC AP	-	-	13	13
		Statistics AP	-	-	-	*
	Total		-	-	39	39
1991-92	Course	Calculus AB AP	-	-	30	30
		Calculus BC AP	-	1	8	9
		Statistics AP	-	-	-	*
	Total		-	1	38	39
1992-93	Course	Calculus AB AP	-	1	26	26
		Calculus BC AP	-	-	11	11
		Statistics AP	-	-	-	*
	Total		-	1	37	38
1993-94	Course	Calculus AB AP	-	-	34	34
		Calculus BC AP	-	-	-	**
		Statistics AP	-	-	-	*
	Total		-	-	34	34
1994-95	Course	Calculus AB AP	-	-	30	30
		Calculus BC AP	-	1	10	11
		Statistics AP	-	-	-	*
	Total		-	1	40	41
1995-96	Course	Calculus AB AP	-	-	37	37
		Calculus BC AP	-	-	-	***
		Statistics AP	-	-	-	*
	Total		-	-	37	37
1996-97	Course	Calculus AB AP	-	1	22	23
		Calculus BC AP	-	1	12	13
		Statistics AP	-	-	-	*
	Total		-	2	34	36
1997-98	Course	Calculus AB AP	-	2	16	18
		Calculus BC AP	-	1	9	10
		Statistics AP	-	1	15	16
	Total		-	4	40	44
1998-99	Course	Calculus AB AP	-	2	10	12
		Calculus BC AP	1	8	10	19
		Statistics AP	1	4	15	20
	Total		2	14	35	51
1999-00	Course	Calculus AB AP	1	1	17	19
		Calculus BC AP	-	13	3	16
		Statistics AP	1	3	48	52
	Total		2	17	39	87
2000-01	Course	Calculus AB AP	-	5	12	17
		Calculus BC AP	-	15	7	22
		Statistics AP	-	5	36	41
	Total		-	25	55	80

- * Advanced Placement Statistics was not offered until 1997-98.
- ** In 1993-94 student transcripts did not differentiate between Calculus AB and Calculus BC.
- *** In 1995-96 too few students enrolled in Calculus BC for the course to be offered.

case previously. A close look at Table 20 indicates that the increase was due to students enrolling in the new AP statistics course. During the hybrid-schedule years, the number of students enrolling in calculus decreased over what had been the case previously. Of those students who studied calculus, a larger percentage completed the two-semester AB/BC sequence than had previously enrolled in the year-long BC course. Another change during the hybrid-schedule years was an increase in the number of eleventh graders enrolling in AP classes.

Beginning in 1999-2000, all Suburban High School students had used a semestered block schedule throughout high school. This was true because seniors that year had been in ninth grade in 1996-97, when the block schedule was piloted with the ninth-grade class. In Table 20, entries for the two “fully block scheduled” years are marked in bold italicized print.

During the fully block scheduled years the number of students enrolling in calculus courses returned to between 35 and 40 students, the level of enrollment that had been typical of the years before the hybrid schedule. The shift towards a larger percentage of calculus students completing Calculus BC continued. In 2000-01 more students completed Calculus BC than had been the case in any previous year.

Meanwhile, a large number of students continued to enroll in the AP statistics class. This led to the total number of students enrolled in AP courses being approximately double what they had been in the years when Suburban High School used a traditional schedule.

It should be noted that, while enrollment at Suburban High School increased slightly at about the time the block schedule was adopted, this change cannot account for the increased enrollment in AP mathematics courses. Between the spring of 1991 and the spring of 1997, the number of students enrolled in AP mathematics divided by the number of students in the senior class generally ranged from .15 to .16. In contrast, in the

spring of 1998 the ratio was .19, in the spring of 1999 the ratio was .23, in the spring of 2000 the ratio was .33, and in the spring of 2001 the ratio was .30.

As noted previously, during the block schedule years there were 38 students who completed both AP calculus and AP statistics. Table 20 records each of these course enrollments as a separate entry adding to the “total” course enrollment reported.

However, even when the enrollment is recalculated to count each student who enrolled in an AP mathematics course only once, regardless of the number of courses in which the student enrolled, the number of students enrolled in AP courses divided by the size of the senior class increased after the semestered block schedule and IMP curriculum were implemented. In the pre-block schedule years the recalculation left the number of students enrolled in AP courses divided by the number of students in the senior class unchanged, at between .15 and .16. In the spring of 1998 the recalculated ratio was .16, in the spring of 1999 the recalculated ratio was .19, in the spring of 2000 the recalculated ratio was .27, and in the spring of 2001 the recalculated ratio was .28.

AP Exam Grades

For a fee, students who have completed an AP course can take an exam administered by the College Board. The exams are graded on the following scale:

- 5: extremely well qualified
- 4: well qualified
- 3: qualified
- 2: possibly qualified
- 1: no recommendation

The College Board sets its grading scale so that the lowest grade of 5 is equivalent

to the average score earned by college students receiving grades of A in college, the lowest grade of 4 is equivalent to the average B, and the lowest grade of 3 is equivalent to the average grade of C. In general, a score of 3 or higher on an AP exam indicates sufficient mastery of course content to allow exemption from a college course, credit, or both. Research has found that when students with AP grades of 3 or better are placed out of introductory college courses, they are more likely to receive grades of A or B in the sequent higher level courses than are those non-AP students who took the introductory course before the higher-level course (College Board, 2001b).

Table 21 reports AP exam grades received by students at Suburban High School each year from 1994-95 through 2000-01. In 1994-95 and 1995-96 Suburban High School paid the fee for AP exams, and required all students enrolled in AP courses to take the exam. In 1994-95 two students who received a course grade lower than a “C” completed the AP exam, so the number of exam takers that year is actually higher than the number successfully completing the course. Key informants at the school weren’t certain of the dates, but believed that the requirement to take the exam was discontinued in 1996-97 and the fee payment was discontinued in 1997-98. For this reason, in 1996-97 and thereafter the number of students taking each exam was noticeably smaller than the number of students successfully completing the corresponding course. It should also be noted that under the semestered block schedule Suburban High School offered Advanced Placement Calculus AB only during the fall semester, while the Advanced Placement exam was not administered until the following May. The current calculus

Table 21. Advanced Placement Exam Grades by School Year and AP Mathematics Course

School Year			Exam Grade					Total (exams completed/ # students completing course)
			5	4	3	2	1	
1994-95	Course	Calculus AB AP	6	15	8	3	1	33/30
		Calculus BC AP	4	2	3	1	-	10/11
		Statistics AP	-	-	-	-	-	*
	Total		10	17	11	4	1	43/41
1995-96	Course	Calculus AB AP	5	11	13	2	3	36/37
		Calculus BC AP	-	-	-	-	-	**
		Statistics AP	-	-	-	-	-	*
	Total		5	11	13	4	3	36/37
1996-97	Course	Calculus AB AP	6	7	2	3	-	18/23
		Calculus BC AP	1	7	3	1	1	13/13
		Statistics AP	-	-	-	-	-	*
	Total		7	14	5	4	1	31/36
1997-98	Course	<i>Calculus AB AP</i>	<i>1</i>	<i>7</i>	<i>2</i>	<i>1</i>	-	<i>11/18</i>
		<i>Calculus BC AP</i>	-	<i>2</i>	-	-	-	<i>2/10</i>
		<i>Statistics AP</i>	<i>2</i>	<i>4</i>	-	-	-	<i>6/16</i>
	Total		<i>3</i>	<i>13</i>	<i>2</i>	<i>1</i>	-	<i>19/44</i>
1998-99	Course	<i>Calculus AB AP</i>	<i>4</i>	<i>6</i>	-	-	-	<i>10/12</i>
		<i>Calculus BC AP</i>	<i>6</i>	<i>1</i>	<i>2</i>	<i>1</i>	-	<i>10/19</i>
		<i>Statistics AP</i>	<i>3</i>	<i>3</i>	-	<i>1</i>	-	<i>7/20</i>
	Total		<i>13</i>	<i>10</i>	<i>2</i>	<i>2</i>	-	<i>27/51</i>
1999-00	Course	<i>Calculus AB AP</i>	<i>2</i>	<i>2</i>	<i>1</i>	-	-	<i>5/19</i>
		<i>Calculus BC AP</i>	<i>12</i>	<i>2</i>	-	-	-	<i>14/16</i>
		<i>Statistics AP</i>	<i>9</i>	<i>6</i>	-	-	-	<i>15/52</i>
	Total		<i>23</i>	<i>10</i>	<i>1</i>	-	-	<i>34/87</i>
2000-01	Course	<i>Calculus AB AP</i>	<i>5</i>	-	<i>1</i>	-	-	<i>6/17</i>
		<i>Calculus BC AP</i>	<i>18</i>	<i>2</i>	-	-	-	<i>20/22</i>
		<i>Statistics AP</i>	<i>10</i>	<i>8</i>	<i>2</i>	-	-	<i>20/41</i>
	Total		<i>33</i>	<i>10</i>	<i>3</i>	-	-	<i>46/80</i>

* Advanced Placement Statistics was not offered until 1997-98..

** In 1995-96 too few students enrolled in Calculus BC for the course to be offered.

teacher at Suburban High School commented that she was concerned that this long delay discouraged a number of students who would otherwise have done so from completing the AB exam.

Table 21 uses the same print convention as Table 20, marking hybrid-schedule cohorts in italicized print and cohorts using a block schedule throughout high school in bold italicized print. The first thing to notice about Table 21 is that although the number of students enrolled in AP courses had increased under the hybrid block schedule (school years 1997-98 and 1998-99) the number of students who went on to take the

exam actually decreased in those years. The number of students taking the tests recovered in 1999-00, and grew to the highest level it had ever been in 2000-01—higher even than in the years when the school paid for the exam and required all students in AP courses to take them.

School year 1999-00 was different from preceding years in two ways that are relevant to the focus of this study. First, 1999-00 was the first year that all students at had been enrolled in a semestered block schedule throughout their years at Suburban High School. Second, a substantial number of the students enrolled in AP courses had utilized IMP as their core curriculum. This is because a number of eleventh graders were enrolled in AP courses, all of whom had used IMP in earlier years. Among twelfth graders that year most students in the Honors program had begun their study of high school mathematics with Algebra 1 in eighth grade and had completed the traditional curriculum in high school.

In school year 2000-01, like the previous year, all students had used a semestered block schedule every year they attended Suburban High School. In addition, except for transfer students all students enrolled in AP courses that year had utilized IMP as their core high school curriculum.

The second thing to notice about Table 21 is the overall improvement in student test scores during the last two years reported in the table, as reflected in the increased number of students receiving a score of “4” or “5” on the exam. These were the two years when the block schedule was fully implemented, and when students taking the AP tests had utilized the IMP curriculum.

The improved performance on AP exams is particularly evident if one focuses on

the most advanced mathematics course offered at Suburban High School: Calculus BC. To make this pattern more evident, Table 22 displays only the Calculus BC AP exam scores.

The Table shows a marked increase in the number of students taking the exam during 1999-00 and 2000-01 and in student exam grades during those years. The improvement in exam grades is larger than can be explained by chance. A logistic regression analysis comparing in the two fully-block-scheduled years (1999-00 and 2000-01) scores to those of the two hybrid-block-scheduled years (1997-98 and 1998-99) yielded a Wald Chi-square statistic of 8.029 with 1 degree of freedom, $p=.005$. A logistic regression analysis comparing the two fully-block-scheduled years to 1996-97, the only traditionally-scheduled year in the table during which students in Advanced Placement Calculus BC had the option of skipping the exam, yielded a Wald Chi-square statistic of 16.3 with 1 degree of freedom, $p<.0005$.

Table 22. Advanced Placement Calculus BC Exam Grades by School Year

School Year	Exam Grade					Total
	5	4	3	2	1	
1994-95 (Traditional Schedule. Traditional Curriculum.)	4	2	3	1	-	10
1995-96 (Traditional Schedule. Traditional Curriculum.)	-	-	-	-	-	*
1996-97 (Traditional Schedule. Traditional Curriculum.)	1	7	3	1	1	13
1997-98 (Hybrid schedule. Traditional Curriculum.)	-	2	-	-	-	2
1998-99 (Hybrid schedule. Traditional Curriculum.)	6	1	2	1	-	10
1999-00 (Block schedule. Mostly IMP)	12	2	-	-	-	14
2000-01 (Block schedule. All IMP.)	18	2	-	-	-	20

Although the previous paragraph labeled 1999-00 and 2000-01 as “fully block scheduled” years, it should be noted that during the two previous “hybrid” years the Calculus BC course had been offered in a two-semester format, just as it was in 1999-00 and 2000-01. A key difference between the years that did so well and the two preceding years is that every Calculus BC student in 2000-01 had used the IMP curriculum, and 13 of the 16 students enrolled in Calculus BC during 1999-00 had used the IMP curriculum. This was the case in 1999-00 because under the semestered block schedule a number of students tended to enroll in Calculus during their junior year. Of the 13 eleventh graders taking Calculus BC in 1999-2000, 12 had previously taken IMP courses. In addition, one of the three twelfth graders taking Calculus BC in 1999-2000 had taken the core IMP sequence. Prior to 1999-00, the only student enrolled in Calculus BC who had utilized the IMP curriculum at Suburban High School was an extremely gifted student who took calculus in tenth grade during 1998-99.

Focusing on the 2000-01 school year, which was the first year in which all students at Suburban High School had used both the IMP curriculum and a semestered block schedule, achievement on the Calculus BC exam was truly impressive. Twenty students took the exam; eighteen received a grade of 5 and the remaining two received a grade of 4, for an average grade of 4.9. To put this in perspective, in 2000-01 there were only eight public schools in the United States where at least 20 students took the Calculus BC exam and the mean grade was 4.9 or better (personal communication, College Board, April 25, 2002).

Chapter 5: Summary and Conclusions

The overall purpose of this study was to investigate the joint impact on mathematics course-taking and high school algebra achievement of a semestered block schedule and a mathematics curriculum designed to conform with the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989). The case of one particular school, Suburban High School, was examined in detail. During the 1996-97 school year, Suburban high school initiated a semestered block schedule for all ninth graders. In that same year, ninth graders were enrolled in IMP, as a phase-in of this new reform-based curriculum.

Summary of Results on the Algebra Achievement Test

In the spring of 1997, eleventh graders at Suburban High school completed a three part Algebra Achievement test. The eleventh graders tested in the spring of 1997 had used a traditional schedule and traditional curriculum throughout Grades 9 through 11. They form a “Traditional” cohort that was contrasted with later “Reform” cohorts.

The testing was repeated in the spring of 2000 and the spring of 2001. Students tested in those years had used a semestered block schedule and the IMP curriculum throughout Grades 9 through 11. They comprise the two “Reform” cohorts who completed the Algebra Achievement test.

The Algebra Achievement test was comprised of three sub-tests. Test 1 measured achievement on understanding and solving applied algebra problems in context. Students had access to graphing calculators while completing Test 1. Test 2 measured achievement on algebra symbol manipulation without context or access to graphing

calculators. Test 3 was completed by students working in pairs. It measured their ability to solve a single, complex, open-ended, applied algebra problem.

Sixth-grade test scores were used as a covariate to control for prior student ability. Optimum Least Squares Linear Regression was used to investigate differences between students in the Reform cohorts and students in the Traditional cohort on Test 1 and Test 2. A Probit analysis was used to investigate differences between students in the Reform cohorts and students in the Traditional cohorts on Test 3. After controlling for prior ability, there is 95% confidence that the following three statements are simultaneously true:

1. On Test 1, the test of understanding and solving applied algebra problems in context, the mean score of students in the Reform cohorts differed from what would have been likely for the same students had they been in the Traditional cohort by between +1.3% and +9.3% correct. In standard deviation terms, this confidence interval was between +0.06 and +0.40 standard deviations. This was a statistically significant difference in favor of the Reform cohorts, as reflected by the fact that the Bonferroni-adjusted confidence interval contains only positive numbers, with the probable impact of being in a Reform cohort ranging from a very small positive effect to a moderate positive effect.
2. On Test 2, the test of symbol manipulation without context or access to graphing calculators, the mean score of students in the Reform cohorts differed from what would have been likely for the same students had they been in the Traditional cohort by between -9.4% and +0.6% correct. In

standard deviation terms, this confidence interval was between -0.36 and $+0.02$ standard deviations. This was a statistically non-significant difference, as reflected by the fact that the Bonferroni-adjusted confidence interval contains both negative and positive numbers, with the probable impact of being in a Reform cohort ranging from a moderate negative effect to a very small positive effect.

3. On Test 3, the test that required pairs of students to solve a single, complex, open-ended, applied algebra problem, the mean achievement of student pairs in the Reform cohorts differed from what would have been likely for the same pairs had they been in the Traditional cohort by between $+0.05$ and $+0.74$ standard deviations. This was a statistically significant difference in favor of the Reform cohorts, as reflected by the fact that the Bonferroni-adjusted confidence interval contains only positive numbers, with the probable impact of being in a Reform cohort ranging from a very small positive effect to a fairly large positive effect.

After controlling for prior ability, the average effect size of being in the Reform cohort on understanding and solving applied algebra problems (Test 1) was $+0.23$ standard deviations, enough to move a student who would ordinarily score at the 50th percentile up to the 59th percentile. The average effect size of being in the Reform cohort on algebraic symbol manipulation problems (Test 2) was -0.17 standard deviations, enough to move a student who would ordinarily score at the 50th percentile down to the 43rd percentile. The average effect size of being in the Reform cohort on a pair's ability to solve an extended open-ended applied algebra problem (Test 3) was $+0.40$ standard

deviations, enough to move a pair who would ordinarily score at the 50th percentile of all pairs up to the 66th percentile.

On Test 1 and Test 2, the effects of being in a Reform cohort appeared to be the same regardless of prior ability. That is, achievement of both high-ability and low-ability students appeared to respond in the same way to membership in the Traditional or Reform cohort. On Test 3, the picture was more complicated. For many pairs of students who took the test, a sixth-grade ability estimate was available for only one member of the pair. When all pairs with any ability estimate were included in the analysis, there appeared to be no interaction with prior ability. When only pairs who had two Grade 6 ability estimates were included in the sample, there did appear to be an interaction, with membership in the Reform cohorts having a more pronounced positive effect on students of lower ability. One possible explanation for the fact that applying differing definitions to measure prior achievement yielded differing results in the test of ability-by-treatment interactions is suggested by the fact that students with a missing measure on Grade 6 achievement were frequently those students who had not attended Suburban High School's feeder middle school. An innovative mathematics curriculum, *Visual Mathematics* (Foreman & Bennett, 1991) was in use at the feeder middle school. An explanation consistent with the data is that either *Visual Mathematics* or IMP was sufficient to enable high-ability students to do well on Test 3, with no value-added effect due to completion of both the visual mathematics and the IMP curricula. However, low-ability students did well only if they had utilized both *Visual Mathematics* and IMP. This explanation must be viewed as an hypothesis to be explored in a later study.

Schoenfeld (2002) noted that data analyzing the achievement effects of curricula designed to implement the *Curriculum and Evaluation Standards* (NCTM, 1989) are just beginning to become available. He stated that the data available so far seem to be converging on the following findings:

1. On tests of basic skills, there are no significant performance differences between students who learn from traditional or reform curricula.
2. On tests of conceptual understanding and problem solving, students who learn from reform curricula consistently out-perform students who learn from traditional curricula by a wide margin.

The results of the Algebra Achievement test were broadly consistent with the findings reported by Schoenfeld, and lend weight to this emerging trend. It should be noted, however, that in the investigation which most closely paralleled the current study, Huntley, et al. (2000) found that while students using a reform curriculum tended to out-perform students using a traditional curriculum on understanding and solving applied algebra problems in context and on solving complex open-ended applied algebra problems, they tended to be less skilled at symbol manipulation without context or access to graphing calculators. In the current study, while the results were not statistically significant, students in the Reform cohorts did perform less well than students in the Traditional cohort on the test of symbol manipulation. A closer look at the specific questions comprising Test 2 indicated that at Suburban High School the advantage of the Traditional cohort on symbol manipulation appeared to be limited to performing well-practiced procedures on items presented in standard form. When items were presented in non-standard form, for example solving $\sqrt{9 + x^2} = 5$ for x , the advantage of being in the Traditional cohort disappeared.

The close look at specific items yielded one cautionary note to the generally positive results on the Algebra Achievement test for students in the Reform cohorts. Fluency in procedures such as translating a line graph to an equation, where students in

the Traditional cohort showed more skill, could be important to student learning in later mathematics or science courses. The new *Principles and Standards for School Mathematics* (NCTM, 2000), published after a decade of debate and consensus building among educators, mathematicians, and parents that followed promulgation of the earlier *Curriculum and Evaluation Standards* (NCTM, 1989) placed increased emphasis on procedural fluency. Further, fluency may aid future learning because effortless processing places fewer demands on conscious attention. Since a person can attend to only a limited amount of information at one time, ease of processing some aspects of a learning task gives the person more capacity to attend to other aspects of the task (Bransford, Brown, & Cocking 1999).

Summary of Results from the Transcript Analysis

The transcript analysis addressed the question, “Did students register in additional or more advanced mathematics under the IMP curriculum and semestered block schedule at Suburban High School than under the prior curriculum and schedule?” Complete transcripts for Grades 9 through 12 of students in the First Reform cohort were compared to complete transcripts for Grades 9 through 12 of students in two earlier cohorts at Suburban High School who had used a traditional schedule and mathematics curriculum throughout high school.

Under the semestered block schedule, more courses were available to students each year than under the traditional schedule, but each class lasted for fewer hours. The net effect of these schedule differences was that about one quarter of the students in the First Reform cohort enrolled in fewer hours of mathematics than would have been likely under a traditional schedule, and about three quarters enrolled in more hours of mathematics than they would have done under a traditional schedule. Of those who enrolled in more hours of mathematics, many enrolled in considerably more.

Students in the First Reform cohort were more likely to enroll in at least one advanced mathematics course than were students using a traditional schedule and curriculum. Further, among students who enrolled in at least one advanced mathematics class, students in the First Reform cohort on average enrolled in more hours of advanced mathematics. This reflects the fact that even though each individual advanced course met for fewer hours under the semestered block schedule, students in the First Reform cohort were more likely to enroll in several advanced mathematics courses than were students using a traditional schedule and curriculum.

Summary of Results Regarding Advanced Placement Courses

As the block schedule and IMP curriculum were implemented, a larger number of students enrolled in AP courses. In part, this was possible because a new AP statistics course was offered under the semestered block schedule. But improved AP participation is not explained by the new statistics course alone. Compared to students in previous years, among students who were enrolled in a semestered block schedule throughout high

and who used IMP as their core mathematics curriculum, a larger number of students enrolled in the most advanced AP mathematics course (Calculus BC), a larger number of students completed the Advanced Placement Calculus BC exam, and their scores on that exam were higher. In 2000-01, the first year in which all students had used the IMP curriculum and all students had been enrolled in a semestered block schedule throughout their years at Suburban High School, twenty students took the Advanced Placement Calculus BC exam. Eighteen of those students achieved a grade of 5 and two students achieved a grade of 4, for an average score of 4.9. That year, there were only seven other public schools in the United States where at least 20 students completed the Advanced Placement Calculus BC exam and achieved an average score of 4.9 or higher.

Interaction of the Semestered Block Schedule and IMP Curriculum

One potential positive effect of a semestered block schedule is to give students and teachers extra time for learning. Less time is spent walking through hallways, and more time is potentially available for studying course content. Nonetheless, previous research has found that when traditional pedagogical techniques and a traditional curriculum are used in the longer time blocks of a block schedule, student achievement in mathematics can suffer (Raphael, Wahlstrom, & McLean, 1986; Marshall, Taylor, Bateson, & Brigden, 1995; Wild, 1998).

The current study has found that at Suburban High School, when a curriculum designed to be consistent with the *Curriculum and Evaluation Standards* (NCTM, 1989) was implemented together with a semestered block schedule, achievement in algebra did not suffer. Students who used the IMP curriculum under a semestered block schedule were more proficient than students who used a traditional curriculum and schedule when asked to formulate mathematical models, to interpretation graphs and tables, or to work in pairs to solve an extended complex algebra problem. Students who used a traditional curriculum and schedule appeared to be advantaged only when applying well-practiced procedures that were presented in standard form.

Results from the Algebra Achievement test indicate that unless “achievement” is defined very narrowly, the semestered block schedule and IMP curriculum together did not harm and in fact improved algebra achievement of students at Suburban High School. Results from AP exam grades corroborate this finding. Students who utilized both the IMP curriculum and semestered block schedule were more likely than others to enroll in AP mathematics, and achieved significantly higher scores when they completed the AP examinations.

The College Board (1998) found that students who complete a two-semester block-length Calculus BC course tend to score higher on the AP exam than do students who complete a traditional all-year course or students who take a one-semester course, so the current results may be partly explained by the two-semester format used under the block schedule. However, AP exam grades of students who took Calculus BC in the two-semester format and had used IMP as their core curriculum were significantly higher than AP exam grades of students who took Calculus BC in the two-semester format and had used a traditional curriculum. Thus, it appears that preparation using the IMP curriculum followed by studying Calculus BC in a two-semester block schedule format may have worked together to greatly improve student achievement.

The transcript analysis provides further evidence that together the semestered block schedule and IMP curriculum provided a considerable benefit to students at Suburban High School. While their algebra achievement by most measures improved, students in the Reform cohorts had considerably more time available to study mathematics topics other than algebra than did students using a traditional schedule and curriculum. In addition to studying the approximately two years worth of algebra and one year's worth of geometry that was completed in the traditional core curriculum, students studied a considerable amount of probability and statistics. Students in Academic Assisted and College Preparatory level courses completed two IMP modules devoted primarily to statistics: *The Game of Pig* and *The Pit and the Pendulum*. Together these modules are equivalent to almost half of a course under a traditional schedule. Students in Honors classes completed these same two modules, plus up to two additional modules that focused on probability and statistics (*Is There Really a Difference?* and *Pennant Fever*). Additional study in probability and statistics is integrated throughout the IMP curriculum. Also, as teachers have become more familiar with the IMP curriculum they have completed a larger number of modules with their students. In future years they expect to complete a larger number of the IMP probability and statistics modules.

Further, students in the Reform cohorts were more likely than students who used a traditional schedule and curriculum to enroll in advanced mathematics beyond the core requirements. These included courses in statistics or AP statistics, discrete mathematics, functional analysis, pre-calculus, Calculus AB and Calculus BC.

In summary, results of this study are consistent with the theory that mathematics achievement will improve at schools that implement a semestered block schedule while simultaneously replacing their core curriculum with the IMP sequence. At Suburban High school, by most measures students' achievement in algebra improved. This was true for both high- and low-ability students. Further, the very top students appeared to be better prepared to take advanced mathematics coursework. Also, while this study focused on algebra achievement and AP test exams, all students had considerably more exposure to probability and statistics than had been true under the traditional curriculum and schedule. Finally, many students had time to take more advanced mathematics coursework beyond the core curriculum than would have been possible under the traditional curriculum and schedule.

Limitations: Threats to Internal Validity

Threats to Internal Validity Attributable to Non-Random Group Assignment.

In a “true experiment” subjects are randomly assigned to treatment and control groups. If assignment is nonrandom, then the research design is referred to as a “quasi-experiment,” and the two groups investigated are commonly referred to as the “treatment” group and the “comparison” group (Krathwohl, 1993).

By this definition, the current study was a quasi-experiment. This increases the risk that there may have been pre-existing differences between students in the Traditional cohort and students in the Reform cohorts at the time they entered ninth grade. The two groups were chosen to be comparable in many ways. They lived in the same community. Many attended the same middle school and used the same middle school mathematics curriculum. They attended the same high school. Due to low staff turnover, they had many of the same teachers both in middle school and high school. Figuratively and in many cases literally, students in the Reform cohorts were the younger brothers and sisters of students in the Traditional cohort.

However, the possibility that some of the results reported here may have been due to a different experience in middle school mathematics class cannot be completely discounted. Both the Traditional cohort and the Reform cohorts used the *Visual Mathematics* curriculum (Foreman & Bennett, 1991) in middle school. However, the Traditional cohort was the first group at the school to use that curriculum. When an innovative program like *Visual Mathematics* is adopted, there is often an “implementation dip” resulting in lower achievement for the first few years (Busick & Inos, 1992; Fullan & Miles, 1992). The Traditional and Reform cohorts had similar scores on mathematics ability in sixth grade, but as Table 2 (see Chapter 3) shows the Traditional cohort had lower scores in seventh grade. As has been discussed in Chapter 3 of this study, this was likely due to a change in the test. Nonetheless, given the possible “implementation dip” the lower scores may have reflected a true difference in what was learned during middle school.

“History” could have affected results in other unknown ways. For example, the size of the junior class increased from 227 in 1996-97 to 279 in 2000-01. Perhaps there were other demographic changes that could have affected test results.

Other Threats to Internal Validity

There were other factors that could have affected achievement and were impossible to control. For example, the individuals teaching both calculus and pre-calculus changed in 1999-00, coincident with the arrival of the first largely-IMP group in the calculus class. Further, Suburban High School adopted a new Calculus textbook, *Calculus: Graphical, Numerical, and Algebraic* (Finney, et al., 1999). The Calculus teacher believed the new text was more in line with the evolving reforms in the AP Calculus curriculum. Together, the change in staff and text provide a possible alternate explanation for some of the success in the AP Calculus program. Various other staffing changes could have affected results on the Algebra Achievement test.

Every effort was made to keep the Algebra Achievement test a low stakes assessment. Teachers at Suburban High school did not review it carefully, and were unlikely to have identified specific topics on the test that they should teach. Nonetheless, the test was administered repeatedly over several years, so the possibility that some teachers unconsciously modified their instruction to accommodate material on the test cannot be completely discounted.

While every effort was made to make testing conditions as comparable as possible for the three administrations of the test, once the block schedule was implemented it was not possible to replicate precisely the testing conditions that had occurred in 1997. Changes made in test administration due to lessons learned during the pilot study partly addressed this problem, but could not overcome it completely. For example, in 1997, students completed the Algebra Achievement test during their mathematics class. In 2000 and 2001, not all students were enrolled in mathematics at the time of testing, so they were administered the test in English classrooms, proctored by mathematics teachers. This meant that pairs completing Test 3 in the Reform cohorts were often from different mathematics classes, whereas in 1997 they had been from the same mathematics class. Due to fears that this difference in pairing might affect results, teachers in 2001 requested that students within each English class pair with someone who had been in their mathematics class. While there is no guarantee that all students complied with this request, many were reported to have done so. Concern that results on Test 3 may have

been due to the way students were assigned to pairs is mitigated by the fact that in 2001, when students were generally paired with partners from the same mathematics class, scores on Test 3 were even higher than in 2000 when no attempt had been made to replicate 1997 testing conditions by matching pairs within a mathematics class.

It should also be noted that only about 90% of eligible eleventh graders completed the Algebra Achievement test each year. Non-participants included some students who were committed to extra-curricular activities that conflicted with test administration times and students who were absent either because of illness or because they elected to skip the test. Had all students taken the test, results might have been different.

As described in Chapter 4, the decision to test students at the end of eleventh grade may have overestimated the achievement of students currently enrolled in mathematics class. A delayed test conducted at the beginning of twelfth grade may have provided a more realistic measure of how much of the algebra students learned by the end of eleventh grade was remembered and available for use in subsequent courses. Thus, the Algebra Achievement test as conducted may have biased results against students using the semestered block schedule, since a portion of them (28.7%) were not enrolled in mathematics class at the time of testing.

Finally, this study measured the overall effects of a semestered block schedule and the IMP curriculum on algebra learning throughout Grades 9-11. Under the Traditional schedule, approximately one-third of the students at Suburban High School enrolled in some form of second-year algebra in twelfth grade. The majority enrolled in Algebra 3/Trigonometry, a course mainly designed to review material they had previously studied, but some students were studying second-year algebra for the first time. Meanwhile, approximately one-third of the students in the Reform cohorts did not complete the core IMP sequence until twelfth grade, and thus had not yet had a full opportunity to learn the algebra content that was tested. Had the testing been conducted at the end of Grade 12, both groups would surely have known more algebra than they did at the end of Grade 11, and the results would have been different in unpredictable ways. It is particularly regrettable that most of the students in the Reform cohorts who were in Academic Assisted classes did not complete the core IMP sequence until twelfth grade, subsequent to the time of testing. Under the new program, Academic Assisted students were offered exposure to more sophisticated mathematics than had previously been the case, and it would have been desirable to measure the full effects of the program on this particular group.

Some Strengths of the Research Design

This study was designed to test the theory that implementing a semestered block schedule together with a *Standards*-based curriculum and adequate professional development and planning time would improve mathematics achievement. The study investigated a falsifiable prediction of that theory. While a research study conducted over several years at a school site is subject to inevitable threats to internal validity, two strengths in the design of the current study deserve mention.

First, there was no “selection bias” of the type that could have been caused by reporting on a site that was known in advance to have implemented a successful intervention. Rather, Suburban High School was selected as the site of this study because it was beginning to implement a program that theory predicted would lead to improved

achievement in mathematics. Data collection began in the spring of 1997, before any results were available to indicate whether or not the implementation would be successful.

Second, the measures of achievement used in this study were not designed with the IMP curriculum in mind. The Algebra Achievement test was designed to accomplish the purposes of this study, that is, to provide a fair yardstick which could compare the algebra achievement of students who had utilized a traditional curriculum to the algebra achievement of students who had utilized a curriculum designed to implement the *Curriculum and Evaluation Standards* (NCTM, 1989). But the test was not designed by the researcher or by anyone associated with the IMP curriculum. Rather, it was designed for a different study (Huntley, et al., 2000) and with a different curriculum in mind, Core-Plus. The secondary test of achievement used in this study, participation rates and scores on the Advanced Placement Calculus BC exam, was of course not designed by the researcher or anyone associated with the IMP curriculum. The other, non-test, measure of achievement reported in this study, student enrollment in mathematics classes and in advanced mathematics classes, was a natural measure of the impact of a reform program that has been used in studying other interventions (Gamoran, Porter, Smithson, & White, 1997).

The Nature of the Implementation at Suburban High School

In technical terms, this section could be entitled “limitations: threats to external validity.” It describes unique conditions at Suburban High School. Other schools that implement a semestered block schedule together with a *Standards*-based curriculum and adequate professional development and planning time will not necessarily get results similar to those that have been described here. The results at other schools might be better or worse because conditions unique to those sites will differ from conditions at Suburban High School. But the unique conditions at Suburban High School provide important lessons to be learned, so the current section provides new insights in addition to limitations.

Interaction of Community and Schedule

Historically, Suburban High School has been a high achieving school. There are many nearby college campuses, and the community puts a high value on education.

Traditionally, 90% of Suburban High School graduates enroll in 2- or 4-year colleges.

One teacher described the situation as follows:

We are a high-end community that values test scores, and math test scores especially. (Before adopting the block schedule) we used to have kids not take lunch in order to take more math and science. After we adopted the block schedule, students took more math. For some students, math became more enjoyable; for others, they wanted to take every mathematics course possible; and others had parents who said, “My child is going to take math every semester so they don’t forget it.”

Adoption of the semestered block schedule at Suburban High School led to a large increase in the number of mathematics classes students took. The school administration increased the minimum number of required classes from three to four, but the vast majority of students had already been taking four classes under the traditional schedule. Under the block schedule, most students increased their course taking to more than was required, enrolling in five or more mathematics courses over their high school career.

The risk that a different community might react very differently to implementation of a semestered block schedule is not merely a theoretical possibility. Gruber and Onwuegbuzie (2001) conducted a similar study in a very different community in rural Georgia, and the results were vastly different. The nearest college to this high school was a 30-mile drive. Although a college preparatory track was available to students, many students on the college preparatory track have traditionally gone on to work or into the military after graduating from high school rather than enrolling in a 2- or 4-year college. Historically this high school required students to complete four units in language arts and three units each in math, science, and social studies in order to graduate. When they adopted a semestered block schedule in 1997-98, the requirements were not changed and the core curriculum was not modified. Teachers continued to use the traditional text they had used previously, covering the same content in each course despite the fewer hours available per course under a semestered block schedule. Also, new academic courses such as statistics or discrete mathematics were not introduced. Students had the option of enrolling in a larger number of academic classes under the semestered block schedule, but they generally did not. Instead, they enrolled in vocational classes, in forestry, or in two semesters of band. The major change in academic course-taking patterns was among students who failed a course. Under the prior scheduling system, those students would have been scheduled to re-register and complete that course again during summer school. Under the semestered block schedule they were able to complete a second enrollment in these same courses during the school year. As a result, the high school eventually eliminated its summer school program (personal communication, authors, April 20, 2002).

Under these conditions, Gruber and Onwuegbuzie (2001) noted that mathematics achievement appeared to suffer after the school adopted a semestered block schedule. Between the spring of 1997 and the spring of 2000, mathematics scores on the Georgia High School Graduation Test fell by a statistically significant .52 standard deviations. These data reflect only three years of the block scheduling, so this drop in tests scores may in part be due to an implementation dip (Busick & Inos, 1992; Fullan & Miles, 1992). Nonetheless, the large decrease is cause for serious concern.

Thus, in communities that differ in academic emphasis and parental education from Suburban High School, adopting a semestered block schedule may not lead to

students enrolling in an increased number of mathematics courses. They may enroll in the same number of courses, with each course lasting fewer hours and covering less content than it had previously. As a result, achievement could suffer.

Schools adopting a semestered block schedule cannot presume that students will enroll in more academic courses in general, and more mathematics courses in particular, simply because more courses are available each year. Instead, existing courses need to be restructured to fit the new schedule. The core curriculum needs to be spread over a larger number of courses, and administrative policies need to be implemented to ensure that students enroll in them. New academic courses need to be created for students wishing to pursue advanced coursework beyond the core curriculum. These things were done at Suburban High School. Such policy changes are probably even more important for schools in less mathematics-oriented communities who are adopting a semestered block schedule.

Planning Time and Professional Development

It was the original intent of this study to investigate the impact of a semestered block schedule and a reform curriculum at a site where teachers had available a considerable amount of planning time and a considerable amount of professional development. This was the case at Suburban High School. It needs to be emphasized that the research literature provides strong evidence that without the planning time and professional development, positive results like those at Suburban High School are unlikely to be realized (Canady & Rettig, 1995; Kramer, 1997a).

Modifications to the IMP Curriculum

Teachers at Suburban High School did not implement the IMP curriculum as intended by its authors. Instead, they made three important changes.

First, they did not follow the IMP authors' recommendation that mathematics be taught in heterogeneous classes. Instead, they maintained three separate "levels" of IMP classes: Academic Assisted, College Preparatory, and Honors. Each level utilized the IMP curriculum, but completed modules at different rates. Over four Integrated Mathematics courses, students in lower-level classes completed less than four complete IMP textbooks. It is the belief of teachers at Suburban High School that despite completing fewer modules, students in the Academic Assisted classes studied much more advanced mathematics than they had done under the traditional program.

Previous research indicates that the approach to ability grouping taken at Suburban High School may be a good one. Gamoran, et al. (1997) studied "transition" courses in California and New York that were designed introduce lower ability students

to college preparatory mathematics, often by teaching the college preparatory content over a longer period of time. They reported that the programs were generally successful. Further, two of the “transition” classes originally in their study had used IMP, but according to the authors the IMP classes were dropped from the study because initial examination of results showed them to be outliers in instruction and achievement. When contacted to provide details about the IMP classes, the authors indicated that test scores in those classes had been sufficiently high so that they skewed the overall results of the study (personal communication, Smithson, 2001).

Despite the support provided by Gamoran et al.’s (1997) article for using IMP as was done at Suburban High School, the authors of IMP have made a strong case for utilizing the curriculum in heterogeneous classes (Alper, et al., 1997). Whether doing so would have affected the results at Suburban High School cannot be answered by the data provided by the current study.

A second change made to the IMP curriculum at Suburban High School may have affected achievement on the Advanced Placement calculus exams. The school continued to use a pre-calculus course, to be taken after completing IMP and before taking calculus. It was the intent of IMP’s authors that students could take calculus immediately after completing the fourth IMP textbook (Green, 2000), or even simultaneously with the fourth textbook (Fendel, Resek, Fraser, & Alper, 1997). Teachers at Suburban High School debated whether completing all four IMP textbooks was sufficient preparation for Calculus. One teacher commented that before studying calculus, students who had completed the traditional core sequence of algebra 1, geometry, and algebra 2 needed an intervening course to reflect on the math they’d learned and see why things are working, to “see the forest for the trees.” Similarly, she felt that after completing the four IMP texts, students still needed that intervening “reflection” course before enrolling in calculus. Other teachers disagreed with the need for such a course. In the end, they decided to keep the pre-calculus course, which utilized the UCSMP text *Precalculus and Discrete Mathematics* (Perissini, et al., 1992). In part, this decision was based on the fact that they could afford to do so, given the extra number of courses students could take under the semestered block schedule. The combination of four IMP courses, followed by pre-calculus, followed by a two-semester sequence of Calculus AB and Calculus BC has certainly achieved spectacular success at Suburban High School. Whether similar success could have been achieved without the intervening pre-calculus course cannot be answered by the current study.

A third modification to the IMP curriculum by the teachers at Suburban High School was the use of complementary materials. This modification is less important than the other two, because the materials were not used extensively for students in the First and Second Reform cohorts. Mathematics teachers at Suburban High School described the development of the complementary materials as follows. For the first two years they were utilizing the IMP curriculum, teachers sometimes made their own work sheets and

used traditional problems as warm ups. In the summer of 1999, teachers responded to the concern of some parents who wanted to see more traditional texts coming home by more formally integrating into their program the algebra 1, geometry, and algebra 2 textbooks that had been in use before the adoption of IMP. They prepared a list of specific problems from the algebra 1, geometry, and algebra 2 textbooks that are matched to specific IMP modules and could be used for enrichment or reinforcement, as needed. Today, these specific problems are sometimes used as homework assignments and sometimes used in class. Because students in the First Reform cohort were entering eleventh grade and students in the Second Reform cohort were entering tenth grade at the time the formal complementary materials were prepared, the materials did not have as much impact on students completing the Algebra Achievement test as they have had on students in subsequent cohorts. Nonetheless, it is possible that achievement results would have been different had these complementary problems not been utilized. Huntley, et al. (2000) provided data indicating that similar complementary materials may have had a positive achievement effect on students utilizing the Core-Plus curriculum.

Visual Mathematics

Most students in either the Traditional cohort or the Reform cohorts had utilized the curriculum Visual Mathematics (Foreman & Bennett, 1991 in middle school. Teachers at Suburban High School believed that the middle school curriculum had a very positive impact on student achievement. The Visual Mathematics curriculum is also an unusually good match to the problem-centered approach used by IMP. Further, there is some indication in the data from Test 3 that previously low-achieving students who had used both the Visual Mathematics curriculum in middle school and the IMP curriculum

in high school showed large improvements in their ability to solve an extended, open-ended algebra problem, but that either curriculum alone was insufficient. Thus, it is possible that results would be different at a high school whose students had experienced more traditional instruction in middle school.

Use of the Visual Mathematics curriculum may have affected not only students' preparation for using IMP, but also the readiness of teachers to implement the IMP curriculum. Before adopting IMP, high school teachers had worked extensively with middle school teachers who had implemented the *Visual Mathematics* curriculum, attending professional development that supported use of the Visual Mathematics curriculum for three consecutive summers. As one Suburban High School mathematics teacher put it, "IMP was easy for most of us to adopt since we were already far along the change continuum by the time we had to make a choice." Thus, it is possible that previous experience with the *Visual Mathematics* middle school curriculum enabled the high school teachers to teach the IMP curriculum more effectively. Results might have been different had this not been the case.

A Strong Grass Roots Leader

Mrs. Sullivan played a key role in changing the mathematics program at Suburban High School. Since 1984, she had served jointly as mathematics supervisor for the school district and as high school mathematics department chair. In that position, she had been involved in hiring new mathematics teachers at both the middle school and the high school and had chosen candidates whose teaching philosophy was broadly compatible with what was eventually outlined in the NCTM *Curriculum and Evaluation Standards* (NCTM, 1989). She arranged high quality professional development over many

consecutive summers, first for the *Visual Mathematics* curriculum and then for the IMP curriculum, and convinced both middle school and high school mathematics teachers to attend the professional development. To build support for the new curricula, she organized parent education nights, and rallied key stakeholders within the community. At various times during the early years IMP was being utilized, she made arrangements to have a reduced teaching load so that she could mentor teachers learning to utilize the new curriculum.

Since leaving Suburban High School in 1999, Mrs. Sullivan has worked with other school districts implementing IMP, Core-Plus, and other curricula designed to implement the *Curriculum and Evaluation Standards* (NCTM, 1989). She believes that those schools who were most successful also had strong grassroots leadership supporting the new program. A Suburban High School IMP mathematics teacher who took a leave of absence from Suburban High School to mentor other schools using new curricula, confirms this opinion. Commenting, based on her experience at Suburban High School and elsewhere about what is need to make a program successful, she noted:

First of all, a grass roots leader in the math department who has a vision and a drive to do the best thing for students regardless of the criticism that inevitably comes from those who do not wish to change. I say grass roots because I think there has to be a leader within the department, a department head or a strong teacher. If it comes from upper administration as a whole it does not seem to work, at least not well or quickly. (Mrs. Sullivan) was the impetus for the change; at first in the middle school, and then as we prepared for Block she led us through the change to IMP.

Unusually Strong Mathematics Teachers

The mathematics teachers at Suburban High School were unusual in a number of ways. They were proactive in designing a mathematics program they believed would benefit their students. Almost all the mathematics teachers at the high school in the mid-

1990s attended professional development to learn about the new middle school curriculum their future students were using, even though they themselves would not be teaching that curriculum. When the high school decided to adopt a block schedule, they searched for a curriculum they thought would match the schedule. Upon identifying the IMP curriculum, they then adapted it to fit the needs of their school and their community. In addition to making the changes described in the previous section, teachers at the school believe they were the first in the nation to adopt IMP school-wide, rather than as an alternative program operating in parallel with a traditional curriculum. Since adopting IMP, most teachers have attended four summers of intensive training in use of the curriculum. The teachers have continued to make changes to their courses, gradually adding modules to course syllabi as they have become more familiar with the program, and adapting and refining the complementary materials they use with the program. It is unclear whether the combination of a semestered block schedule and a Standards-based curriculum could have been successful at a school with a less assertive and professional group of mathematics teachers.

Collegial Mathematics Department

Both the principal and mathematics teachers at Suburban High School emphasized the fact that a major factor in the mathematics department's success was the ability of teachers in the department to work well together. Other schools trying to replicate the results of the current study may need to pay attention not only to developing the skills of individual mathematics teachers, but also to developing a collegial atmosphere in which mathematics teachers can build on one another's strengths.

Time to Prepare for the Change

After the high school administration decided to adopt a semestered block schedule, and before it was implemented, teachers at the school were given a year to prepare for the change. One teacher described the importance of this planning year as follows:

We were given a year to prepare for the change and our department, with Mrs. Sullivan's direction, did do our homework. Every in-service day was spent collaborating about what do we want our students to be able to do mathematically and how are we going to get them there. Then we looked at the curriculum that was available and chose IMP.

Without a year to plan, teachers at Suburban High School may not have been able to make the changes that appear to have contributed to the success of their mathematics program under a semestered block schedule.

Elements of Community Support

With strong support from the district superintendent and assistant superintendent, Mrs. Sullivan had implemented the Visual Mathematics curriculum at the middle school, beginning in 1991-92. Subsequently, both the superintendent and assistant superintendent left the district, so most of the impetus for change at the high school came from Mrs. Sullivan and other teachers there. By the time they began to consider IMP, a series of parent nights and similar events had organized a degree of community support for the Visual Mathematics curriculum. Further, mathematics test scores had improved at the middle school, and some community members perceived the improvement as being due to the use of Visual Mathematics. Finally, when the high school faculty was considering how to adapt their mathematics curriculum to meet the needs of a semestered block schedule, Mrs. Sullivan arranged a meeting with a group of mathematics professors from nearby colleges who were also parents of children attending the high school. This group strongly endorsed the adoption of an integrated curriculum,

and it was a member of this college group who first suggested that the high school consider utilizing IMP. The support of an influential group of local parents with strong education credentials helped counter vocal opposition to the new program that was expressed by other groups within the community.

Use of Multiple Measures

This study has confirmed the repeated finding of many investigations going back at least to the NLSMA reports prepared by the School Mathematics Study Group in the 1960s: Mathematics achievement is a multidimensional phenomenon. More recent research comparing student learning under traditional mathematics instruction to student learning under Standards-based mathematics instruction has found that the results can vary, depending on what type of test is used to measure achievement (Wood & Sellers, 1996; Huntley, et al., 2000). Tests emphasizing procedures and symbol manipulation tend to favor traditional instruction, whereas tests emphasizing problem solving in context tend to favor reform instruction. This means that studies reporting changes in “mathematics achievement” can be highly misleading, unless they provide a detailed description of precisely what was tested in their measure of “achievement”. It is possible to construct a test that will favor almost any program, so long as the test covers only the specific skills at which that program excels.

The current study used an assessment containing three very different measures of mathematics achievement and found that results differed depending on the measure. Further, considerable insight into what students did and did not learn was provided by a detailed examination of student responses to specific questions within each of the measures. Given the controversy that has surrounded reform mathematics programs like

IMP, it is particularly important for researchers reporting results to explain just what they measured, and whenever possible to use multiple measures that will assess a range of student abilities.

Synergy of IMP and the Semestered Block Schedule

The current study was designed to investigate the joint effects of a semestered block schedule and the IMP curriculum, at a site where sufficient planning time and professional development were available to make it likely these changes would be implemented effectively. Results of the study cannot be attributed to either the curriculum or schedule alone. Rather, it is the impact of both together that appears to have achieved positive results.

Regarding a semestered block schedule, there is considerable evidence based on anecdotes, observations, and surveys that successful implementation requires changes in teaching style, changes in curriculum, and changes in the support given to teachers. The majority of achievement research to date has been conducted at sites where few if any of these changes have been made. The result has usually been equivocal or negative effects on mathematics achievement.

This is the first study that investigated mathematics achievement a site at which all of the research-recommended changes had been implemented. While the mathematics teachers at Suburban High School were engaged in extensive planning to adapt their program to the needs of the semestered block schedule even before they decided to utilize IMP, the adoption of IMP made it much easier for them to modify course content and instruction in a way that fit the new schedule. One possible reason so much other research has been conducted at sites that did not make the recommended changes when

adopting a semestered block schedule is that, absent a reform curriculum, such changes may be difficult or impossible to implement.

The result of the implementation was a very positive effect on student mathematics learning. Students in the Reform cohorts had a somewhat different, but generally more flexible and stronger profile of algebra knowledge than did students in the Traditional cohort. Students had more opportunity to study probability and statistics, to study other mathematics beyond the core curriculum, and to enroll in Advanced Placement classes. The two-semester course in Calculus BC that was possible under a semestered block schedule was particularly successful, as measured by student performance on the AP Calculus BC examination.

Regarding IMP, there has been considerable controversy about the worth of that curriculum. In particular, Wu (2000) and others have accused the curriculum of inadequately preparing students to study advanced mathematics. They have labeled the program as providing insufficient drill, insufficient support for the teacher in guiding students to abstraction and generalization of ideas, and insufficient emphasis on mathematical precision. In contrast, the authors view the curriculum's problem-centered approach as a likely vehicle for empowering students as mathematical thinkers with a deep understanding of underlying concepts (Alper, et al., 1997). In particular, the curriculum might help to overcome the problem of "inert knowledge" described by Bransford, et al. (1989), and thus might better prepare students for future learning.

As implemented at Suburban High School, there was some limited evidence that the curriculum's lack of drill resulted in less automatic or fluent operation on procedures, as Wu (2000) predicted. However, the advantage students in the Traditional cohort had

in performing procedural operations appeared to be “inert.” When symbol manipulation problems were presented in anything other than the standard format, students in the Reform cohorts were as likely to solve them as were students in the Traditional cohort.

Students who used the IMP curriculum had higher achievement on items requiring formulation of mathematical models, interpretation of tables and graphs, and cooperative work with a partner to solve an extended, open-ended problem involving linear equations. It should be noted that the success of learning algebra under IMP may have been due in part to the longer time blocks under a semestered block schedule, which could perhaps support IMP’s style of investigations.

Students who had utilized the IMP curriculum seemed particularly well prepared for studying calculus. After studying Calculus BC under a two-semester format they scored significantly higher on the Advanced Calculus BC exam than had students who used a traditional curriculum and studied Calculus BC under the same format. Although alternate explanations cannot be ruled out, this result is consistent with the hypothesis that algebra concepts might be less “inert” if they are learned using IMP materials instead of a traditional curriculum.

As can be seen from the previous paragraph, this study provides direct evidence regarding the validity of concerns about IMP’s ability to provide students a firm technical foundation for pursuit of more advanced work in mathematics or science (Wu, 2000). The experience at Suburban High School provides strong evidence that students who complete the IMP curriculum can have an excellent technical foundation to pursue advanced study in mathematics or science.

It should be noted, however, that after completing the IMP sequence students at Suburban High School did complete a pre-calculus course before enrolling in calculus, just as students using the traditional core sequence had done. Some teachers at Suburban High School believed students would not have been as successful at calculus without the intervening course. Given that the IMP sequence uses four texts, versus three for the traditional sequence, without the extra courses available under a semestered block schedule it would have been difficult or impossible for many students to have completed IMP, then completed pre-calculus, and still have had time for calculus. Whether forgoing the pre-calculus course would have negatively affected students' achievement in calculus cannot be determined from the current study.

Mrs. Sullivan, the former mathematics supervisor at Suburban High School noted that one of the goals teachers hoped to achieve by adopting a reform curriculum was “to get our mainstream kids, not just our top level kids, to enroll in advanced mathematics courses.” There is some research evidence that utilizing the IMP curriculum can indeed motivate students who wouldn't otherwise do so to study advanced mathematics (Webb, in press). While the IMP curriculum may have motivated some students to enroll in advanced courses, the semestered block schedule provided extra opportunities for them to do so. Again, the current study cannot separate the effects of the two interventions. Together, they did accomplish the goal of increasing mainstream students' participation in advanced mathematics classes. Overall, the proportion of students enrolling in at least one advanced course beyond the core curriculum increased from 48% in traditional cohorts to 58% in the First Reform cohort. This was true despite the fact that students in the First Reform cohort had to complete four core courses before they were able to study

more advanced mathematics, compared to three core courses for students in the traditional cohorts. Further, the proportion of students studying Advanced Placement classes swelled, due mainly to the large number of students interested in Advanced Placements statistics. About 13% of the students in the First Reform cohort enrolled in the Advanced Placement statistics course.

According to one study, the success of teachers at Suburban High School in encouraging mainstream students to study advanced mathematics courses may have had a strong impact on their students' future. An investigation of longitudinal data from the High School and Beyond survey reported that one of the strongest predictors of whether an individual would successfully earn a bachelors is the highest level of mathematics he or she completed in high school. 62% of students who completed trigonometry in high school went on to completed a bachelor's degree, versus only 39.5% of students whose highest course was algebra 2 (Adelman, 1999).

Future Research

There is more research to be done at Suburban High School. When they were in twelfth grade, some students in the Traditional cohort completed two blocks of items from the National Assessment of Educational Progress (NAEP) focusing on geometry and on probability and statistics. Students in the First Reform cohort completed the same items, and students in the Second Reform cohort are scheduled to do so this year. These items will be scored and analyzed, to provide a picture of differences between cohorts in knowledge of geometry and probability and statistics. Also, standardized tests administered by the state in which Suburban High School resides will be analyzed, to provide another view of how the curriculum and schedule have affected achievement.

It is more important, however, to extend the current study to other and contrasting sites. A particularly useful study would be to implement a similar program at an historically under-achieving high school. Block schedules, when implemented correctly, are purported to have particularly positive effects on schools with discipline problems, and to be particularly helpful for low achieving students (Kramer, 1997a). Further, there is general agreement both among proponents of the IMP curriculum and among those who have opposed some of its implementations that IMP is likely to be particularly effective for students who have historically been disaffected from mathematics (Alper, et al., 1997; Wu, 2000).

Most importantly, the experience at Suburban High School has demonstrated that a semestered block schedule, when properly implemented, can provide students the opportunity to take a larger number of advanced mathematics courses than they would otherwise. When implemented together with the IMP curriculum, previously reported negative side effects of a semestered block schedule on student mathematics achievement were prevented and even reversed.

Adelman (1999) in reviewing data from the High School and Beyond longitudinal study found that the highest level mathematics course completed in high school was a strong predictor of whether or not a student would eventually be able to complete a bachelor's degree successfully. Adelman's study was correlational so it is unclear whether providing a student with more opportunity to study advanced mathematics will truly improve his or her prospects later in life but it is a possibility. Given the current emphasis on educational opportunity for all, as reflected in the such documents as the *Principle and Standards for School Mathematics* (NCTM, 2000) and the *No Child Left*

Behind Act of 2001 (U. S. Department of Education, 2002) such a possibility is certainly worth pursuing. Implementing a semestered block schedule together with IMP or a similar curriculum is a promising vehicle for increasing all students' opportunity to study advanced mathematics.

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Appendix A: Core-Plus Algebra Test Forms Used In This Study

Part 1: Understanding and Solving Algebra Problems in Context

Reasoning with Algebra and Functions

Part 1

Form C

Student Name _____

Teacher Name _____

* * * * *

Many problems encountered in business, science, and government require mathematical analysis of relations among several variable quantities. Ideas and methods of algebra and functions are often useful in reasoning about those problems.

The following pages contain 4 problem sets. Each set involves questions about relationships among variables. *For each question write what you believe is the best answer and show the work and reasoning that support your answer.*

If you use a graphing calculator in your work on a question, please explain *briefly* the way you used it (e.g. "I graphed $y = 3x + 2$ and traced looking for $y = 12$ " or "I made a table for $y = x^2$ and looked for x that made $y = 81$.")

You have 40 minutes for the 4 problem sets, so don't spend too long on any one problem or question if you get stuck.

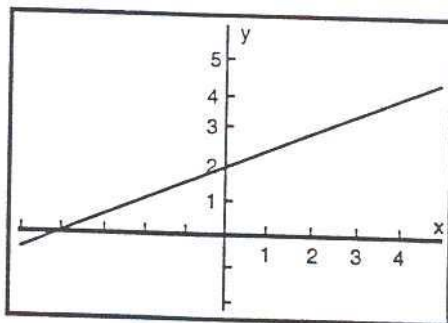
Problem 1: Writing Relationships in Algebraic Form

One of the most important tasks in using methods of algebra to solve problems is finding a way to express problem conditions in symbolic form—function rules, equations, inequalities, etc. That often means translating numerical data, graphs, or written words and sentences into symbolic expressions.

Questions 1.1 - 1.5 give information about relationships between variables and ask you to express that information in equations or function rules.

Question 1.1: The *Watchdog Security Service* provides home and business security systems with installation charge of \$150 and a charge of \$5 per week to monitor the system. Write a formula that gives cost C of the service as a function of time t in weeks that the service is used.

Question 1.2: Write an equation relating x and y that will produce this graph.

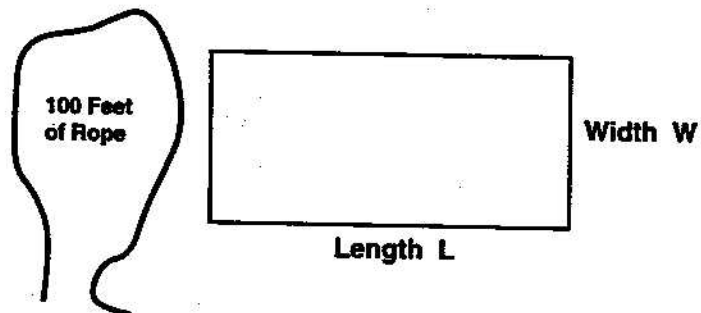


Question 1.3: Pedro and Maria ride a school bus 15 miles to high school. Write an equation that shows how to calculate the average speed S (in miles per hour) of their trip from the time T (in hours) that the trip takes.

Question 1.4: Write an equation relating x and y that will give the pairs of numbers in this table.

x	-1	0	1	2	3
y	0.5	1	2	4	8

Question 1.5: In a school science project each student is given 100 feet of rope to enclose an experimental garden plot in the shape of a rectangle.



- a) Write an equation showing how the *width* W of any such plot depends on the *length* L .

$$W = \underline{\hspace{2cm}}$$

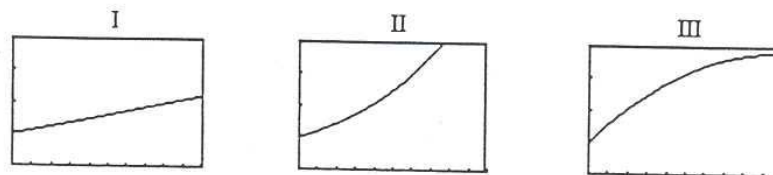
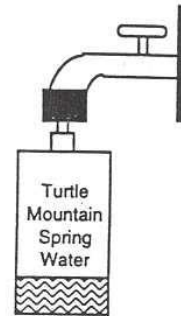
- b) Write another equation showing how the *area* of any plot also depends only on the *length*.

$$A = \underline{\hspace{2cm}}$$

Problem 2: Tapping the Market

Recent reports about bacteria and parasites in city drinking water supplies have led to a booming business in sales of bottled water and water purifying devices. Water has become a big business.

The following graphs show three different predictions of how income from water sales might change over the next several years.



Use the graphs to answer questions 2.1 and 2.2 about water business prospects.

Question 2.1: Which of the three graphs (I, II, or III) shows annual income rising:

- a) at an increasing rate? _____
- b) at a constant rate? _____
- c) at a decreasing rate? _____

Question 2.2: Income from sales of bottled water in 1990 was \$500 million. What is the best match between the three graphs and these equations relating sales income S in millions of dollars to time t in years since 1990?

- a) $S = 500 + 60t$ _____
- b) $S = -15t^2 + 290t + 500$ _____
- c) $S = 500(1.2)^t$ _____

The Turtle Mountain Spring Company made plans about growth in their own share of the water business. They predicted that annual income from sale of their bottled water B and filters F would change over time according to the following formulas. Time t is in years since 1990 and income is in millions of dollars per year.

$$\text{Bottled Water Income: } B = 20 + 5t$$

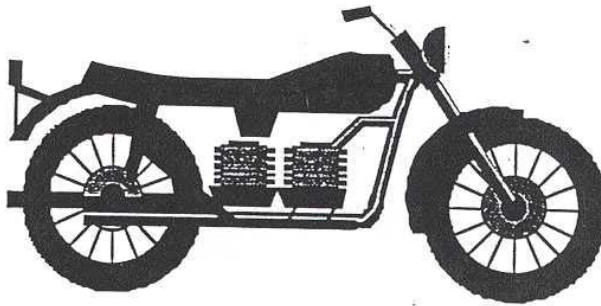
$$\text{Filtering Devices Income: } F = 28 + 3t$$

Use those formulas to answer questions 2.3 - 2.5. Remember to show your work and explain briefly how (if at all) you used a graphing calculator in your work.

Question 2.3: When is the income from bottled water projected to reach \$50 million?

Question 2.4: When does the Turtle Mountain Springs Company expect the two water products to give the same annual income?

Problem 3: The Price Has To Be Right



The builders of a new American Eagle motorcycle plan to set a fixed price for the cycle—no rebates or bargaining allowed. The problem is finding the right price to charge. They estimate that their operating *costs* c and *sales income* I will be related to the *number* n of cycles that are made and sold:

$$\text{Operating Costs: } c = 800n + 10,032,000$$

$$\text{Income from Sales: } I = 4500n - 0.2n^2$$

Answer each of the following questions that arise in studying business prospects for American Eagle's planned new motorcycle. Remember to show your work and explain briefly how (if at all) you used a graphing calculator.

Question 3.1: What will the operating cost be if 5000 cycles are made and sold?

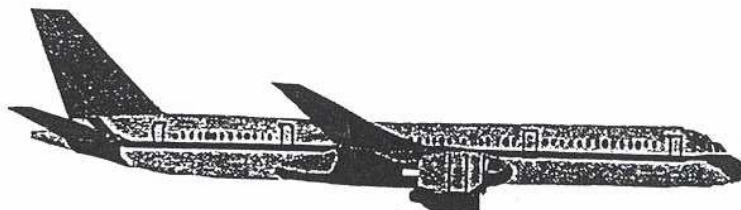
Question 3.2: What will the income from sales be if 2000 cycles are sold?

Question 3.3: For what number of cycle sales will income be at least \$9,112,500?



Question 3.4: For what number of cycle sales will income be as large as possible, and what will that income be?

Problem 4: The Long-Distance Airliner



Several commercial airlines have non-stop flights from Los Angeles, California all the way to Sydney, Australia. It is a trip of 7500 miles and can take as long as 18 hours, most of the time out over the Pacific Ocean. Therefore, estimating flight time and fuel requirements is very important. One airline uses this formula to calculate flight time T in hours from expected wind speed W in miles per hour:

$$T = \frac{7500}{500 + W}$$

Explain briefly what each of the following algebraic calculations will tell about flight plans. You do not need to do the calculations.

Question 4.1: What will you learn from calculating T when $W = 75$?

Question 4.2: What will you learn from solving: $\frac{7500}{500 + W} = 14$?

Question 4.3: What will you learn from solving: $\frac{7500}{500 + W} > 14$?

The *fuel* F in gallons required for a flight is a function of estimated *time* T in hours and number of *passengers* N on the flight. Those variables are related by the equation:

$$F = 2500T + 1.5NT + 1500$$

*Explain briefly what each of the following algebraic calculations will tell about flight plans.
You do not need to do the calculations.*

Question 4.4: What will you learn from calculating F when $T = 14$ and $N = 225$?

Question 4.5: What will you learn by solving $2500T + 1.5NT + 1500 < 48,000$ for N when $T = 16$?

Part 2: Symbol Manipulation

Reasoning with Algebra and Functions

Part 2

Form A

Student Name _____

Teacher Name _____

* * * * *

On this section of the algebra/functions assessment there are 14 questions. You have 20 minutes to answer them. The questions require algebraic operations that are simple enough to do without use of calculators or computers. **Calculators and computers are not to be used on this section of the assessment.**

Section 2.1

Questions 1-6 are in multiple-choice format. For each question, circle the letter of the choice that you believe is the **best** answer. Use the blank space for scratch work in your reasoning.

1. Which of the following expressions is equivalent to $(4xy^3)(5y)$?

- (a) xy^2
- (b) $9xy^3$
- (c) $9xy^4$
- (d) $20xy^3$
- (e) $20xy^4$

2. Which of the following expressions is equivalent to $\frac{125 + x}{25}$?

- (a) $5x$
- (b) $5 + x$
- (c) $100 + x$
- (d) $\frac{126x}{25}$
- (e) $5 + \frac{x}{25}$

3. Which of the following inequalities is equivalent to $4x - 7 > 3$?

- (a) $x > -1$
- (b) $x > 0$
- (c) $x > \frac{7}{4}$
- (d) $x > \frac{10}{4}$
- (e) $x > 3$

4. If $t \neq 0$, which of the following expressions is equivalent to $\frac{8}{t} - \frac{5}{2t}$?
- (a) $\frac{3}{3t}$
 - (b) $\frac{11}{2t}$
 - (c) $\frac{3}{2t^2}$
 - (d) $\frac{13}{2t^2}$
 - (e) $\frac{3}{-t}$
5. A rectangle has length L and width W . Its length is 4 meters greater than its width. Which of the following describes the relation between L and W ?
- (a) $W = L + 4$
 - (b) $L = W + 4$
 - (c) $W > L + 4$
 - (d) $L + 4 > W$
 - (e) $L > W + 4$
6. Which of the following expressions is equivalent to $-3(5x - 6)$?
- (a) $-15x + 18$
 - (b) $2x - 9$
 - (c) $-15x - 6$
 - (d) $-15x - 18$
 - (e) $2x + 3$

Section 2.2

Give answers for the 8 questions in this section. Use the work space to show your reasoning. Record your results in the "answer" boxes.

7. If $s = 2$ and $t = -3$, then what is $2st + (s^2 t)$?

Work Space:

Answer:

8. Solve the equation $3x + 4 = 5x - 2$ for x .

Work Space:

Answer:

9. Solve the system of equations
$$\begin{cases} -2x + 3y = 8 \\ x - y = 2 \end{cases}$$

Work Space:

Answer:

10. Solve the equation $x^2 + 3x - 4 = 0$ for x .

Work Space:

Answer:

11. Solve $(2x - 6)(x + 4) = 0$ for x .

Work Space:

Answer:

12. Solve $4 = \frac{15}{x+3}$ for x .

Work Space:

Answer:

Part 3: Extended, Applied Algebra Problem

Reasoning with Algebra and Functions

Part 3

Form A

Student Name(s) _____

Teacher Name _____

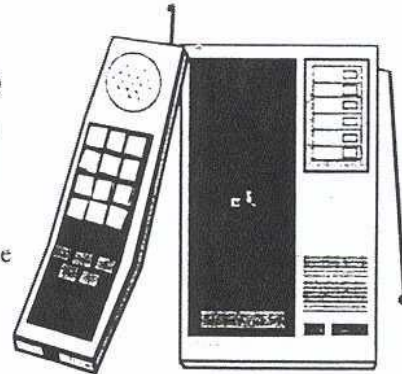
This section of the algebra/functions assessment contains a single problem, based on several pieces of given information. Use the given facts and your mathematical ideas and skills to “solve” the problem.

You have 20 minutes to develop a response working with a partner. You may use a calculator and any other resources that are available for use in your regular homework or classwork assignments. Some graph paper is attached to the question page.

Make notes of ideas and calculations as you work on the problem. *Then combine your ideas to write a clear and complete report of your conclusions, explaining how you arrived at your results, including a brief explanation of how (if at all) you used a graphing calculator.*

Selling by Telephone

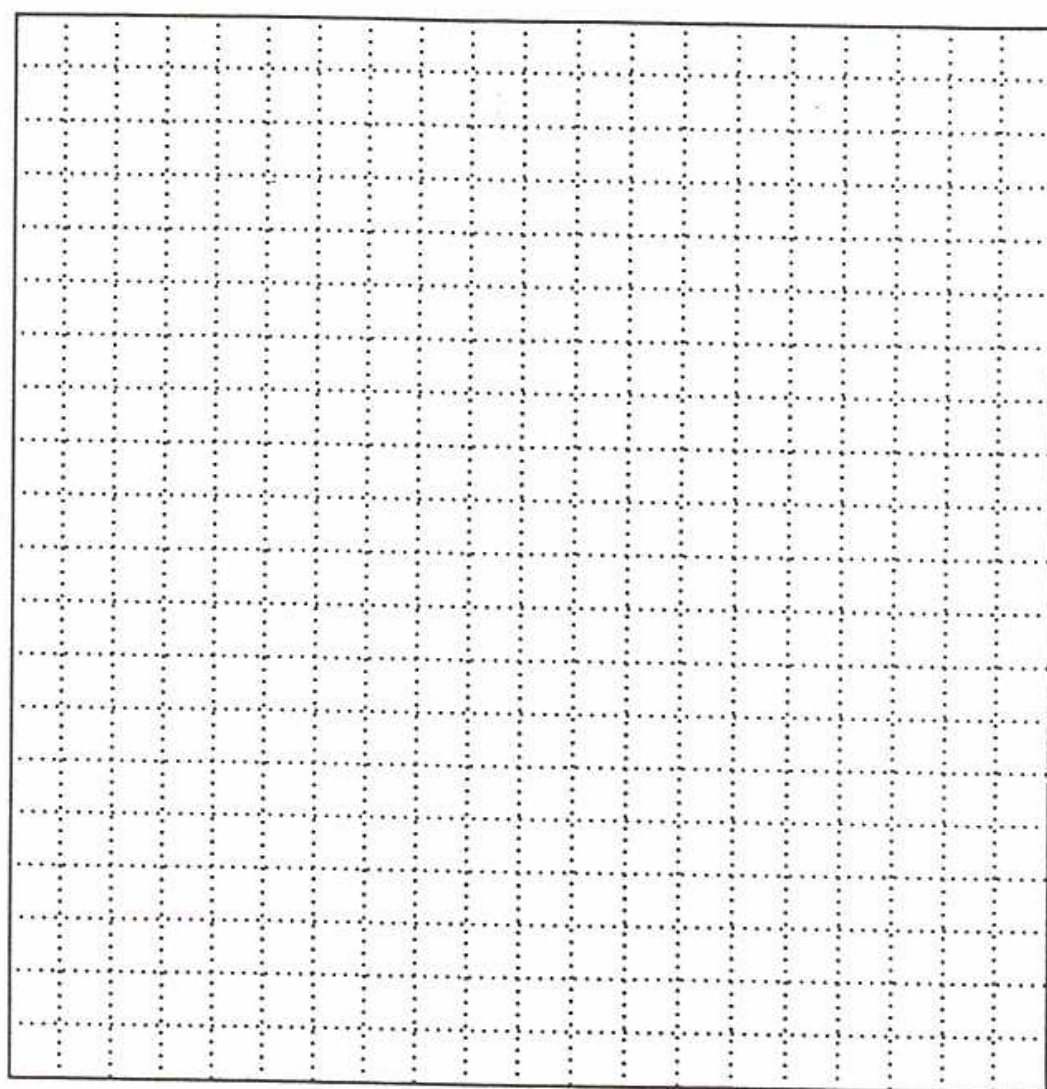
Many companies sell their products through long-distance telephone calls to customers. For example, the *CD Club* sells music compact discs all across the country from its headquarters in New York. Sales calls made by the *CD Club* last an average of about 4 minutes apiece. The club has bids from three possible providers for their long-distance telephone service:



- 1) *Apple Communications* will charge \$0.35 for placing each call and then \$0.15 per minute of time used in the call.
- 2) *Bell Telephone* makes a fixed charge of \$260 per week for access to its long-distance lines, but charges only \$0.10 per minute of time used by the calls.
- 3) *Capital Long Distance Services* will make a fixed charge of \$600 per week for unlimited use of its long-distance lines.

Question: For the *CD Club* the problem is to choose the long-distance service that is least expensive for their business. What advice would you give about the phone company to choose and why?

Conclusions and Reasoning:



Appendix B: Scoring Rubrics

The attached documents are photocopies of the rubrics actually used for scoring. The original rubrics were handwritten by the authors of the Algebra Achievement test. Prior to the training session for scorers who participated in the current study, the current author made a number of changes to the rubrics so that they addressed unanticipated responses in some of the student work analyzed by Huntley, et al. (2000). As noted in Chapter 3, the most extensive changes were made to the rubric for Test 3. Further changes to the rubrics were penciled-in during the training session for scorers who participated in the current study. These changes documented guidance and clarifications given in response to scorers' questions about how to apply the rubrics reliably to anchor and sample papers. Subsequent to the initial training session, the rubrics were not further modified. At that time, rubric contents were fixed in order to ensure that all student papers used in this study were scored to the same standard.

Guiding Principles for Scoring Exams

Guiding Principles for Scoring Exams

General Grading Scheme

- Full credit is given if the answer is correct, regardless of the work shown. (exception 3.1, 3.2)
- Partial credit is given when there is evidence (i.e., supporting work) of progress toward the solution. If there is no supporting work shown, partial credit is given in cases where an incorrect answer is obviously obtained from a mental calculation or from using a calculator, and indicates correct thinking but a minor error in execution.
- Minor errors (i.e., arithmetic errors) receive a small deduction if the problem is beefy, but receive a large deduction if the problem is not very involved.
- Major errors (i.e., algebraic errors such as passing errors, combining unlike powers of x) receive a large deduction.
- Students are rewarded for their correct work, even if their further work toward solution to a problem indicates a major error(s).
- Half points are given sparingly.
- If students provide two different answers to a problem (even if one answer is correct), no credit is given.

Part 1

- 2 The answer is correct or essentially correct (i.e., all of the critical elements of the solution are present, but there is evidence of a transcription or small arithmetic error).
- 1.5 The answer is essentially correct, but some critical element of the solution is omitted.
(NOTE: This category is not very broad.)
- 1 The answer indicates evidence of progress toward the solution, or at least one major step toward solution is correct.
or
If the problem is not very involved, a small arithmetic error may receive a score of 1.
(NOTE: This category is very broad.)
- 0 The answer contains major errors, or there is no progress toward solution, or the question is left blank.

Part 2

Section 2.1 (multiple-choice; no partial credit)

- 2 — correct answer
- 1 — incorrect answer

Section 2.2 (open-response items; no problem context)

- 3 — correct answer (regardless of work)
- 2 — minor (i.e., arithmetic) error
- 1 — significant (i.e., algebraic) error(s)
- 0 — many errors or blank

Part 3

- 4 Contains a complete response with clear, coherent, unambiguous explanation; includes clear and simple graphs and/or diagrams, if appropriate; communicates effectively to an identified audience; shows understanding of the question's mathematical ideas and processes; identifies all the important elements of the question.
- 3 Contains a good solid response with some, but not all, of the characteristics above.
- 2 Contains a complete response, but the explanation is muddled; presents incomplete arguments; includes graphs and/or diagrams that are inappropriate or unclear or fails to provide a graph or diagram when it would be appropriate; indicates some understanding of the mathematical ideas, but in an unclear way.
- 1 Omits parts or all of the question and response; has major errors; uses inappropriate strategies.
- 0 No response or glib response with no apparent thought or effort.

Rubric for Part 1

UPPATER Rubric For Part 1

Question 1.1: The *Watchdog Security Service* provides home and business security systems with installation charge of \$150 and a charge of \$5 per week to monitor the system. Write a formula that gives cost C of the service as a function of time t in weeks that the service is used.

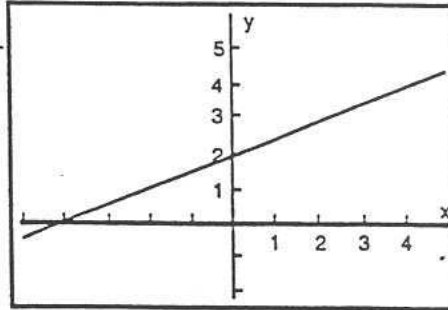
$$2 \quad \left[\begin{array}{l} C = 150 + 5t \text{ or } 150 + 5t \\ \text{Next} = \text{Now} + 5 \text{ [Start 150]} \end{array} \right.$$

$$1 \quad \left[\begin{array}{l} C = 5t \\ \text{Next} = \text{Now} + 5 \text{ (no I.C.)} \end{array} \right.$$

Question 1.2: Write an equation relating x and y that will produce this graph.

2 $y = \frac{1}{2}x + 2$ or equiv

1 $\left\{ \begin{array}{l} \text{slope \& y-int but} \\ \text{no equation or} \\ y = \frac{1}{2}x + b \quad (b \neq 2) \\ \text{or} \\ y = mx + 2 \quad (m \neq \frac{1}{2}) \\ \text{or} \\ \frac{1}{2}x + 2 \quad (\text{no } y =) \end{array} \right.$



0 $\left\{ \begin{array}{l} \frac{1}{2}x + b \quad (\text{no } y =) \\ \text{or} \\ mx + 2 \quad (\text{no } y =) \end{array} \right.$ or worse.