

Question 1.3: Pedro and Maria ride a school bus 15 miles to high school. Write an equation that shows how to calculate the average speed S (in miles per hour) of their trip from the time T (in hours) that the trip takes.

2 $S = \frac{15}{T}$ or $TS = 15$ or equiv

1 $S = \frac{D}{T}$ or equiv. or $\frac{15}{t}$ so $S =$

Page - 2

○ $S = 15T$ or $T = 15S$ or other
wrong relation

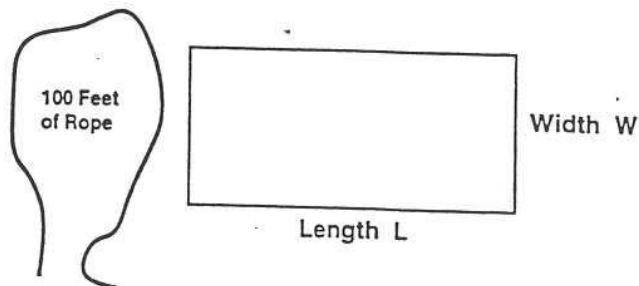
Question 1.4: Write an equation relating x and y that will give the pairs of numbers in this table.

x	-1	0	1	2	3
y	0.5	1	2	4	8

2 $y = 2^x$ or
 Next = 2 · Now [start 1]
 or
 "IF $x=0$, $y=1$ and if x goes up 1, y doubles"

1 $\left[\begin{array}{l} \text{Next} = 2 \cdot \text{Now} \quad (\text{no i.c.}) \\ 2^x \quad ("y=" \text{omitted}) \\ \text{or} \\ \text{"IF } x \text{ goes up 1, } y \text{ doubles"} \text{ or equiv. no start value} \end{array} \right.$

Question 1.5: In a school science project each student is given 100 feet of rope to enclose an experimental garden plot in the shape of a rectangle.



- a) Write an equation showing how the width W of any such plot depends on the length L .

2 $W = \frac{50-L}{1}$ or $\frac{100-2L}{2}$

1 $2W + 2L = 100$ or $\frac{50-2L}{100-L}$ $\frac{2L-100}{2}$ or equiv.

$W = 100 - 2L/2$ (where scorer can't tell whether 2 was to be divided into both terms one algebraic error in solving $2W + 2L = 100$, or other work shown.)

0 $W = A/L$ or incorrect Algebraic expression from a or more errors

But note: students are rewarded for their correct work, even if their further work towards solution of a problem indicates major errors.

Student writes
" $L = 50$
 $W = 50$ "
and scorer computes
 $2 \times 50 + 2 \times 50 = 100$
0.5
 $W = A/L$

1.5

- b) Write another equation showing how the *area* of any plot also depends only on the *length*.

2 $A = \underline{L(50-L)} \text{ or } L \cdot (\text{expression obtained in (a)})$

1 $\begin{cases} A = L \cdot W \text{ or} \\ A = B \cdot H \end{cases}$



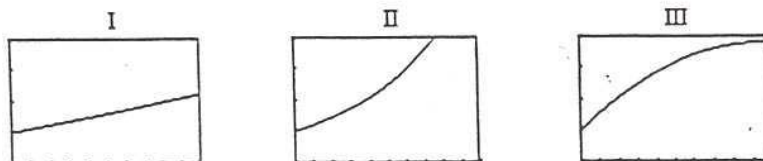
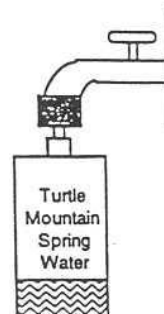
~~except~~

$A = L \cdot \frac{A}{L}$ is
a t best
all

Problem 2: Tapping the Market

Recent reports about bacteria and parasites in city drinking water supplies have led to a booming business in sales of bottled water and water purifying devices. Water has become a big business.

The following graphs show three different predictions of how income from water sales might change over the next several years.



Use the graphs to answer questions 2.1 and 2.2 about water business prospects.

Question 2.1: Which of the three graphs (I, II, or III) shows annual income rising:

- | | | |
|---------------------------|------------|------------------------|
| a) at an increasing rate? | <u>II</u> | 2 - all O.K. |
| b) at a constant rate? | <u>I</u> | 1 - one or two matches |
| c) at a decreasing rate? | <u>III</u> | 0 - no matches |

Question 2.2: Income from sales of bottled water in 1990 was \$500 million. What is the best match between the three graphs and these equations relating sales income S in millions of dollars to time t in years since 1990?

- | | | |
|------------------------------|------------|------------------------|
| a) $S = 500 + 60t$ | <u>I</u> | 2 - all O.K. |
| b) $S = -15t^2 + 290t + 500$ | <u>III</u> | 1 - one or two matches |
| c) $S = 500(1.2)^t$ | <u>II</u> | 0 - no matches |

The Turtle Mountain Spring Company made plans about growth in their own share of the water business. They predicted that annual income from sale of their bottled water B and filters F would change over time according to the following formulas. Time t is in years since 1990 and income is in millions of dollars per year.

$$\text{Bottled Water Income: } B = 20 + 5t$$

$$\text{Filtering Devices Income: } F = 28 + 3t$$

Use those formulas to answer questions 2.3 and 2.4. Remember to show your work and explain briefly how (if at all) you used a graphing calculator in your work.

Question 2.3: When is the income from bottled water projected to reach \$50 million?

- 2 1996 or after 6 years or $t=6$
- 1.5 correct except for arith. error or minor graphing error
- 1 $50 = 20 + 5t$ but no further work or algebra error
 or $20 + 5t = 50,000,000$ with complete, correct algebraic work
 e.g. $t = 9,999,996$
- 0.5 $20 + 5t = 50,000,000$ with no further work
- 0 ~~None of the above~~ Anything Worse.

2 : correct AND Explanation work shown $20 + 5t = 50$
 or graphed the equation
 or both

1.5 correct, no explanation

OR
 correct, except for arith. error

The Turtle Mountain Spring Company made plans about growth in their own share of the water business. They predicted that annual income from sale of their bottled water B and filters F would change over time according to the following formulas. Time t is in years since 1990 and income is in millions of dollars per year.

$$\text{Bottled Water Income: } B = 20 + 5t$$

$$\text{Filtering Devices Income: } F = 28 + 3t$$

Use those formulas to answer questions 2.3 and 2.4. Remember to show your work and explain briefly how (if at all) you used a graphing calculator in your work.

Question 2.4: When does the Turtle Mountain Springs Company expect the two water products to give the same annual income?

2 1994 or after 4 years or $t=4$ w/ explanation

1.5 correct except for arithmetic error or minor graphing
or correct, no explanation

1 explicit appropriate graphing or graphing-calculator strategy with ~~major error~~ ~~wrong answer~~ (e.g. describes graphing both equations and looking for intersection, but writes y-coordinate (40) as the answer)

or

$20 + 5t = 28 + 3t$ but no further work or algebra error

The builders of a new American Eagle motorcycle plan to set a fixed price for the cycle—no rebates or bargaining allowed. The problem is finding the right price to charge. They estimate that their operating costs c and sales income I will be related to the number n of cycles that are made and sold:

$$\text{Operating Costs: } c = 800n + 10,032,000$$

$$\text{Income from Sales: } I = 4500n - 0.2n^2$$

Answer each of the following questions that arise in studying business prospects for American Eagle's planned new motorcycle. Remember to show your work and explain briefly how (if at all) you used a graphing calculator.

Question 3.2: What will the income from sales be if 2000 cycles are sold?

2 8,300,000

2 ~~1.5~~ 1 DISCUSS Should 8.2.E6 be a ~~"P.5"~~ No. a "2"?
 8.2 E6 = 82,000,000 ~~"P.5"~~
 Subst OK (4500 · 2000 - 0.2 · 2000²) + arith error
OR Substitution alone
OR
 17,500,000 (derived from substituting 5000 correctly into equation.)
~~Discuss Should 8.2.E6 = 82,000,000 be a "1"?~~
 wrong subst even w/ correct arith.
 OR 4500(2000) - 0.2(4000)

82,000,000 by itself is a "0"

0- Says "substitute", but does not show it, and gets wrong answer.

The builders of a new American Eagle motorcycle plan to set a fixed price for the cycle—no rebates or bargaining allowed. The problem is finding the right price to charge. They estimate that their operating costs c and sales income I will be related to the number n of cycles that are made and sold:

$$\text{Operating Costs: } c = 800n + 10,032,000$$

$$\text{Income from Sales: } I = 4500n - 0.2n^2$$

Answer each of the following questions that arise in studying business prospects for American Eagle's planned new motorcycle. Remember to show your work and explain briefly how (if at all) you used a graphing calculator.

Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

Note: for problem 3.3, a "reasonable approximation" is ± 50 of the exact value

2 | $2250 \leq n \leq 20250$ or
inequality w/ reasonable approx to 2250 and 20250 $\rightarrow 2.0$

1.5 | $n \leq 20250$ (or reasonable approx) or
 $n \geq 2250$ " " " or $\rightarrow 1.0$

1.5 | $n = 2250, 20250$ (both values) (or reasonable approx) $\rightarrow 1.5$

5 | $n = 2250$ or $n = 20250$ (only one value) (or reasonable approx)
with $4500(2250) - 0.2(2250)^2$ is 9,112,500

5 | $9,112,500 \geq 4500n - 0.2n^2$
finds one value within range e.g. 11,000

as an
but negativity, it is a "0"

note

$9,112,500 > 4500n - 0.2n^2$ is also a .5

The builders of a new American Eagle motorcycle plan to set a fixed price for the cycle—no rebates or bargaining allowed. The problem is finding the right price to charge. They estimate that their operating costs c and sales income I will be related to the number n of cycles that are made and sold:

$$\text{Operating Costs: } c = 800n + 10,032,000$$

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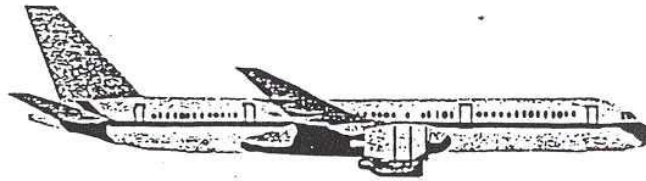
Answer each of the following questions that arise in studying business prospects for American Eagle's planned new motorcycle. Remember to show your work and explain briefly how (if at all) you used a graphing calculator.

Question 3.4: For what number of cycle sales will income be as large as possible, and what will that income be?

2 $n = 11,250 (\pm 1250)$ and $I = 25,312,500$, or close approximation
 1.5 one of answers, w/ explanation.
 note: $[25,000,000 \text{ to } 25,400,000]$ is close

1 $n = 11,250 (\pm 1250)$ or I in interval $[25,000,000 \text{ to } 25,400,000]$
or
 correct strategy explained

Problem 4: The Long-Distance Airliner



Several commercial airlines have non-stop flights from Los Angeles, California all the way to Sydney, Australia. It is a trip of 7500 miles and can take as long as 18 hours, most of the time out over the Pacific Ocean. Therefore, estimating flight time and fuel requirements is very important. One airline uses this formula to calculate flight time T in hours from expected wind speed W in miles per hour:

$$T = \frac{7500}{500 + W}$$

Explain briefly what each of the following algebraic calculations will tell about flight plans.

You do not need to do the calculations.

Question 4.1: What will you learn from calculating T when $W = 75$?

2 Time for trip (in hours - optional) when wind speed is 75 (mph - optional)

1.5 Correct statement relating time for trip (in hrs - optional) to wind speed (mph optional). No mention of 75 mph

e.g., "time for trip according to wind speed" or "time for trip decreases as W increases"

1 { Time for trip (in optional hrs)
Time for trip (in optional hrs) when travelling at 75 mph
Time for trip is 13 (hrs) or $T = 13.04$ (hrs)

0 { 13.04 OR $T = x$, $x \neq 13$ or 13.04 (arithmetic error)
or
incorrect statement (e.g., "time for trip increases as W increases")

Several commercial airlines have non-stop flights from Los Angeles, California all the way to Sydney, Australia. It is a trip of 7500 miles and can take as long as 18 hours, most of the time out over the Pacific Ocean. Therefore, estimating flight time and fuel requirements is very important. One airline uses this formula to calculate flight time T in hours from expected wind speed W in miles per hour:

$$T = \frac{7500}{500 + W}$$

Explain briefly what each of the following algebraic calculations will tell about flight plans.

You do not need to do the calculations.

Question 4.2: What will you learn from solving: $\frac{7500}{500 + W} = 14$?

2 Wind speed for a 14 hour trip.

1 [Wind speed (in mph) or equivalent
 $W = 35.7$ or $W = 36$ or $W = 35$

0 [35.7 or 36 or 35 or $W = x, x \neq 35.7, 36, 35$
 (arith. error)
 or
 "wind time" or "the wind" or other terminology
 that does not connote "speed"

note that you must have
 windspeed to get any
 credit at all.

(i.e. "trip of 14 hrs" is
 not enough by itself to
 get credit.)

Several commercial airlines have non-stop flights from Los Angeles, California all the way to Sydney, Australia. It is a trip of 7500 miles and can take as long as 18 hours, most of the time out over the Pacific Ocean. Therefore, estimating flight time and fuel requirements is very important. One airline uses this formula to calculate flight time T in hours from expected wind speed W in miles per hour:

$$T = \frac{7500}{500 + W}$$

Explain briefly what each of the following algebraic calculations will tell about flight plans.

You do not need to do the calculations.

Question 4.3: What will you learn from solving: $\frac{7500}{500 + W} > 14$?

2 What wind speeds will cause a trip longer than 14 (hrs - optional)

1 Trip time of more than 14 hrs or
 $W < 35.7$, etc or

"You will learn what wind speed is less than" etc

"What wind speed will cause a trip less than 14"

explains $= 14$ or ≤ 14 instead of > 14 and doesn't mention windspeed
 or $W = 35.7$ etc or $W > 35.7$

or
 > 14 , but time for trip (or equiv.) missing

or
 $W < \text{something}$ (Algebraic solution incomplete, "windspeed" not explicit)

or
 "windspeed" but less than something (or equivalent) missing

The fuel F in gallons required for a flight is a function of estimated time T in hours and number of passengers N on the flight. Those variables are related by the equation:

$$F = 2500T + 1.5NT + 1500$$

Explain briefly what each of the following algebraic calculations will tell about flight plans.
You do not need to do the calculations.

Question 4.4: What will you learn from calculating F when $T = 14$ and $N = 225$?

2- Fuel required (in g - optional) for a trip of 14 hrs w/ 225 passengers.

5- Omits one of "14 hrs" or "225 passengers"

1- Fuel required or $F = 41,225$

3- 41,225 OR $F = x$, $x \neq 41,225$

note: to score above 0, the answer must have either "fuel required" (or equivalent), or else " $F = 41,225$ ".

1.5 ~~Score can infer that~~ student describes
Fuel as a function
of # hrs and # passengers
but doesn't specifically mention
"14 hrs," or "225 passengers"

The fuel F in gallons required for a flight is a function of estimated time T in hours and number of passengers N on the flight. Those variables are related by the equation:

$$F = 2500T + 1.5NT + 1500$$

Explain briefly what each of the following algebraic calculations will tell about flight plans.

You do not need to do the calculations.

Question 4.5: What will you learn by solving $2500T + 1.5NT + 1500 < 48,000$ for N when $T = 16$?

- 2 Possible number of passengers on a trip of 16 hrs using less than 48,000 g of fuel. (or equivalent)
- or
- Maximum number of passengers on a 16-hr trip with 48,000 g of fuel. (or equivalent)
- 1 Omits "16 hrs" or doesn't ~~indicate~~ indicate an inequality.
- or
- ~~Omits both "less than" 48,000 and "maximum number passengers"~~
- or
- $n < 270.8$ or $n < 270$ [correct algebraic solution]
- 0 Omits "less than 48,000 g of fuel" or "how many passengers"
- or
- incorrect algebraic solution

Rubric for Part 2

Revised - modified rubric

Changes: added "N" on
problem 10
problem 13
problem 14

Reasoning with Algebra and Functions

Part 2

Form A

~~6/3/99~~
final

Student Name Possible Score Key

Teacher Name Graders: M, J (C)

On this section of the algebra/functions assessment there are 14 questions. You have 20 minutes to answer them. The questions require algebraic operations that are simple enough to do without use of calculators or computers. Calculators and computers are not to be used on this section of the assessment.

Section 2.1

2 = right answer
0 = wrong answer

Section 2.2

OR
see details
inside

3: right answer (regardless of work)
2: minor error (arithmetic error)
OR 1 of 2 correct answers of system of linear eqs
1: significant error
OR 1 of 2 correct answers of a quadratic eq
0: lot of error
N: blank, or aside answer with absolutely no
attempt to answer problem

Section 2.1

Questions 1-6 are in multiple-choice format. For each question, circle the letter of the choice that you believe is the best answer. Use the blank space for scratch work in your reasoning.

1. Which of the following expressions is equivalent to $(4xy^3)(5y)$?

(a) xy^2
(b) $9xy^3$
(c) $9xy^4$
(d) $20xy^3$
(e) $20xy^4$

2. Which of the following expressions is equivalent to $\frac{125+x}{25}$?

(a) $5x$
(b) $5+x$
(c) $100+x$
(d) $\frac{126x}{25}$
(e) $5+\frac{x}{25}$

3. Which of the following inequalities is equivalent to $4x-7>3$?

(a) $x > -1$
(b) $x > 0$
(c) $x > \frac{7}{4}$
(d) $x > \frac{10}{4}$
(e) $x > 3$

4. If $t \neq 0$, which of the following expressions is equivalent to $\frac{8}{t} - \frac{5}{2t}$?

(a) $\frac{3}{3t}$

(b) $\frac{11}{2t}$

(c) $\frac{3}{2t^2}$

(d) $\frac{13}{2t^2}$

(e) $\frac{3}{-t}$

5. A rectangle has length L and width W . Its length is 4 meters greater than its width. Which of the following describes the relation between L and W ?

(a) $W = L + 4$

(b) $L = W + 4$

(c) $W > L + 4$

(d) $L + 4 > W$

(e) $L > W + 4$

6. Which of the following expressions is equivalent to $-3(5x - 6)$?

(a) $-15x + 18$

(b) $2x - 9$

(c) $-15x - 6$

(d) $-15x - 18$

(e) $2x + 3$

7.
1 pt. - correct
alone

i-subs. alone
one church
~~sp. sp. alone?~~

wiparen

3: -24
2: correct sub + correct rate of each item e.g. $-12 + -12 = 24$ c: 0
 $-12 \cdot -12 = 144$

Answer: -24

Answer: -24

Work Space:

Answer:
 $x = 3$

Answer:
 $x = 3$

ex. $3x - 5x = -2 \cdot 4 \rightarrow -2x = -8 \rightarrow x = 4$

or some other variant.

do several steps, at least one step correct,
have several mistakes:

Work Space:

Answer:
 $x = 14$ & $y = 12$

$$\begin{array}{rcl} 2: & -2x + 3y & = 8 \\ & 3x - 3y & = 2 \\ & x & = 1 \\ & y & = 9 \end{array}$$

OR make substitution
minor mistake at arith

$y = 8$

1. eliminate equations get one correct equation in one variable.
isolate one variable " " " " ~~two variables~~

2.

(0 pts) $-x + 2y = 10$

10. Solve the equation $x^2 + 3x - 4 = 0$ for x .

Work Space:

Answer:

$$x = -4 \text{ and } 1$$

3. -4 and 1 (If both are present, student does not need to show work.)

2. $(x+4)(x-1) \rightarrow (4,1) \text{ or } (-4,-1) \text{ or } (4,-1)$ [i.e. everything correct except the sign]

Note: to score at least a "2", student should know that a quadratic equation must have two answers.

1. $(x+4)(x-1) = 0$ and no answer or wrong answer
(e.g. just "1" as an answer)

OR

incorrect factor & get logical answer, e.g. $(x-4)(x+1) \rightarrow x = 4, -1$

OR

-4 or 1 (just one answer) with reasonable justification

e.g. factoring, or substituting " $1 + 3 - 4 = 0$, so answer is 1",
or " $16 + 12 - 4 = 0$, so answer is -4."

0: lots of errors ~~scribbles~~ -4 or 1 (one answer only), with unreasonable justification, or no justification.

e.g. incorrectly combines " $x^2 + 3x$ " to get " $4x$ " or " $4x^2$ ", or " $4x^3$ ", and reasons something like

$$x^2 + 3x - 4 = 0$$

$$4x^2 - 4 = 0$$

$$4x^2 = 4$$

$$x^2 = 1$$

$$x = 1$$

11. Solve $(2x-6)(x+4)=0$ for x .

Work Space:

3: 3 and -4

2: $(x+4)=0$, $(2x-6)=0$ have some arith. errors.

1: 3 or -4 OR expand and use quadratic formula

0: $2x^2 + 2x - 24$

Answer:

$x = 3$ and -4

12. Solve $4 = \frac{15}{x+3}$ for x .

Work Space:

3: $\frac{3}{4}$

2: $4x + 12 = 15$ and some arith. errors

1: $4x + 12 = 15$ and no further work

OR ~~$4x + 3 = 15$~~ $4x + 3 = 15 \rightarrow x = 3$

0: lots of error or blank. OR $4 = 5 + x \rightarrow x = -1$

Answer:

$x = 3/4$

13. Solve $\sqrt{9+x^2} = 5$ for x .

Work Space:

Answer:

$x = 4$ and -4

3: 4 and -4 (If both are present, student does not need to show work.)

2: minor arithmetic error (This score is very uncommon.)

1: 4 or -4 with appropriate work

Example: substitute in equation, e.g. $\sqrt{9+4^2} = 5$

or show that they were trying to compute $\sqrt{16}$, or $9+x^2=25$

0: 4 or -4 with no evidence of appropriate reasoning

or

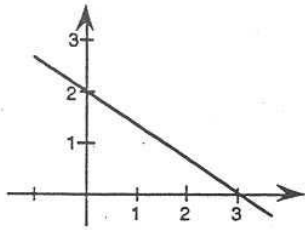
$$3+x=5 \rightarrow x=2$$

14. What is an equation of the line shown in the graph below?

Work Space:

Answer:

$$y = -\frac{2}{3}x + 2$$



1.5 $y = -\frac{2}{3}x + 2$ [or] $3y + 2x = 6$ or equivalent

OR leaves off "y=" (everything else correct), e.g. $-\frac{2}{3}x + 2$

OR incorrect y-intercept (everything else correct), e.g. $y = -\frac{2}{3}x + b, b \neq 2$

OR get correct slope ($-\frac{2}{3}$) and correct y-intercept (2), but no equation

OR $y = \frac{2}{3}x + 2$

1.0 $y = mx + 2, m \neq \pm \frac{2}{3}$ [i.e. incorrect slope, but everything else correct]

OR leaves off "y=" and has incorrect y-intercept, but slope appears correct

e.g. $\begin{cases} -\frac{2}{3}x + b, b \neq 2 \\ "2x + 3y" \end{cases}$

OR slope = $-\frac{2}{3}$ OR y-intercept = 2

OR $y = -\frac{2}{3} + 2$, if it seems clear which is slope & which is y-intercept

OR $\frac{2}{3}x + 2$ ("no 'y='", incorrect sign on slope)

OR leaves off "y=" and has incorrect slope, even if y-intercept appears correct (i.e., $mx + 2, m \neq \pm \frac{2}{3}$)

OR $3x + 2y = 6$ or any other constant (or other similar error)
e.g., $3x + 2y = 0$

OR $y = 2 - \frac{2}{3}$, where it seems unclear which is slope & which is intercept

UPDATED

There are three parts to a correct and complete response to this question.

A) Correct model:

Students obtain the following linear models relating "Number of Calls" and "Cost":

Apple:	$C = .35N + 4(0.15)N = .95N$
Bell:	$C = 260 + 4(0.10)N = 260 + 0.40N$
Capital:	$C = 600$

As long as the scorers can be certain that students used the correct model, they get credit for having done so, even if they did not communicate it clearly. Moreover, if students have not clearly explained their model, but have the correct answer, (which they could not have obtained without using a correct model), they get credit for a correct model and a correct answer, but not for a correct explanation.

B) Consistent answer: Students communicate their answer clearly and unambiguously. The answer must correctly apply the model they used, although for scores of "3" or lower, slight rounding errors due to use of a table or graphic solution method are acceptable. (reasonable approximation counts.)

If students used the correct model, the "consistent answer" required would be:

If 472 or fewer calls are made, then the CD club should choose Apple.

If 473-850 calls are made, then the CD Club should choose Bell.

If 850 calls are made, then the CD Club should choose Capital.

See appendix II for common "close but wrong" equations, and the associated "consistent" answers.

C) Correct explanation: Students describe in detail how they obtained their solution, with enough explanation to justify the choices they made in identifying the cheapest service provider. While a graph or a table of values are not absolutely required, providing one is likely to shore up weak points in an explanation that would otherwise be inadequate. Completely correct, correctly labeled graph is enough. (needs company names)

SCORING

4 - all three elements (model, answer, and explanation) are correct and complete. (intersection points)

3 - two of the three elements are correct and complete.

2 - one of the three elements is correct and complete.

1 - Type A: With few exceptions--described in Appendix III-- students who analyze which company is best by assuming the CD club makes a particular number of calls (or uses for a particular number of minutes) receive a "1" (but add plausible explanation for your "points") = 2

- Type B: at least one of the three elements (model, answer, and

~~2 discrete~~ 2 discrete points
Correct & complete
no plausible explanation is a "0"

2 discrete points with plausible explanation is a "3"

explanation) was seriously attempted, but no element is correct and complete.

0 - no response or glib response with no serious attempt to model or address the question. This includes "nonsense number combinations", as well as inadequate informal models, or assertions with no evidence of mathematical analysis (e.g. "Choose Capital because of the fixed price.")

a discrete case not done completely
correctly is a "0" → but add
plausible explanation
for your "put" and it
becomes a
(1)

Appendix I: Details for choosing between a 1 (“serious attempt”) and a 0

minimum for serious attempt at “model”: The students present enough evidence so the scorer can confidently infer that they used one of the “close but wrong” models described in Appendix II, or a similar model. It is NOT sufficient to present a model for only one company (e.g., writing “Apple costs \$.95 per call”).

minimum for serious attempt at “explanation”: It is very uncommon to have a sufficient explanation for a “1”, while having a totally inadequate model and a totally inadequate answer. Students could conceivably do so if they presented enough information so the scorer could confidently infer that, had they followed the procedure they describe and used the correct model, the students could have gotten a correct answer.

minimum for serious attempt at an “answer”: The students describe qualitatively the idea that for very few calls the CD club should choose Apple, for a moderate number of calls the CD club should choose Bell, and for a large number of calls the CD club should choose Capital. They may even make up intervals that present this flavor (e.g., merely writing “0-50 calls choose Apple; 51-220 calls choose Bell; 221 or more choose Capital”). However, an informal model that does less than this (e.g., writing only “choose Capital if you’re making lots of calls”) scores a “0”.

Appendix II: Commonly Seen Linear Equations

Correct equations.

$$\begin{array}{ll}\text{Apple:} & C = .35N + 4(0.15)N = .95N \\ \text{Bell:} & C = 260 + 4(0.10)N = 260 + 0.40N \\ \text{Capital:} & C = 600\end{array}$$

If 472 or fewer calls are made, then the CD club should choose Apple.

If 473-850 calls are made, then the CD Club should choose Bell.

If 850 calls are made, then the CD Club should choose Capital.

An equivalent model in terms of "minutes":

$$\begin{array}{ll}\text{Apple:} & C = .35(1/4)M + 0.15N = 0.2375M \\ \text{Bell:} & C = 260 + 0.10M = 260 + 0.10M \\ \text{Capital:} & C = 600\end{array}$$

If 1888 minutes or fewer of calls are made, then the CD club should choose Apple.

If 1892-3400 minutes of calls are made, then the CD Club should choose Bell.

If 3400 or more minutes of calls are made, then the CD Club should choose Capital.)

"Close but wrong" linear equations

$$\begin{array}{ll}\text{Apple:} & C = .35x + 0.15x \\ \text{Bell:} & C = 260 + 0.1x \\ \text{Capital:} & C = 600\end{array}$$

Choose Apple for less than 650 minutes;

Bell for 650-3400 minutes;

Capital for more than 3400 minutes.

$$\begin{array}{ll}\text{Apple:} & C = .35 + 0.15x \\ \text{Bell:} & C = 260 + 0.1x \\ \text{Capital:} & C = 600\end{array}$$

Choose Apple for less than 3998 minutes;

Capital for more than 3998 minutes.

$$\begin{array}{ll}\text{Apple:} & C = .35x + 0.15(4) = .35x + 0.6 \\ \text{Bell:} & C = 260 + 0.1(4)x = 260 + .4x \\ \text{Capital:} & C = 600\end{array}$$

Choose Apple for up to 1712 calls;

Capital for more than 1712 calls.

Apple: $C = .35 + 0.15x$

Bell: $C = 260 + 0.1x$

Capital: $C = 600$

Choose Apple for less than 4000 minutes;
Capital for 4000 or more minutes.

Apple: $C = .35 + 0.15(4)x = .35 + 0.6x$

Bell: $C = 260 + 0.1x$

Capital: $C = 600$

Choose Apple for up to 519 minutes or calls;
Bell for 520-3400 minutes or calls;
Capital for more than 3400 minutes or calls.

Apple: $C = .35x + 0.15(4)x = .95x$

Bell: $C = 260 + 0.1x$

Capital: $C = 600$

Choose Apple for up to 305 calls;
Bell for 306-3400 minutes or calls;
Capital for more than 3400 minutes or calls.

Appendix III: Analysis at one or more discrete values.

It is very difficult for students to score better than a "1" if they present their answer at one or more discrete values (i.e., for a particular number of minutes or a particular number of calls) rather than over a range. (An example of "analysis at discrete values" would be the following answer:

"Choose Apple. If you make one phone call it will only cost you .95, whereas Bell would cost \$260.40 and Capital would cost \$600.")

~~If the analysis is done at a discrete value, the "model" is automatically incorrect.~~
Generally, if a correct or close-but-wrong set of linear equations is applied to get the cost for each of the three service providers at a discrete value, the students will have made a sufficiently serious attempt at modeling to score a "1"--even if students made minor arithmetic errors in the computation. HOWEVER, failing to compute the cost for Bell at the discrete value, or failing to compute the cost for Apple at the discrete value would further reduce the score from "1" to "0" (i.e., the model would no longer count as a "serious attempt").

If analyzed at a discrete value, the **explanation** is correct (i.e., can raise the score from a "1" to a "2") only if the following elements are present: The choice of the discrete value must be justified, and the justification must be convincing and reasonable. All computations must be made clearly and correctly, and the cost for each long distance provider at the discrete value must be provided. Moreover, this cost must be computed correctly, using the correct linear equations (not one of the "close but incorrect" models).

Example of a correct explanation when analyzing a discrete value:

"If the CD club sells compact discs all across the country, it must have at least one full-time sales person. One full-time sales person working a 40 hour week would complete 40×60 divided by 4 = 600 phone calls per week. At 600 calls per week, the cost for Apple Communications is \$.95 per call $\times$ 600 = \$570 per week. The cost for Bell Telephone is \$260 per week + \$.40 $\times$ 600 calls = \$500 per week. The cost for Capital is \$600. Therefore, we'd choose Bell."

If analyzed at discrete values, the **answer** is correct (i.e., can raise the score from a "1" to a "2") only if the following elements are present: the case of low, medium, and high number of calls are all considered, or else a clear and convincing justification must be given for why all three cases are not considered. At each discrete value, the cost for each of the long distance service providers is presented, and the costs are correct (not merely "consistent" with one of the "close but incorrect" models). At each discrete value, the least expensive long distance provider is correctly identified.

Example of a correct answer when analyzing discrete values:

"For only 2 calls, Apple long distance would cost \$1.90, Bell would cost \$260.80, and Capital would cost \$600. At 600 calls, Apple would cost \$570, Bell would cost \$500.

Capital, which costs \$600 would be the most expensive until the CD club made lots of calls --e.g., at 1000, Apple would cost \$950 and Bell would cost \$660."

It is possible--but unlikely-- in the discrete case to get both the **answer** and the **explanation** correct, thus scoring a "3". An example is the following extension of the correct "explanation" above, which includes justification for why cases other than 1000 are not considered:

"If the CD club sells compact discs all across the country, it must have several full-time sales people. One full-time sales person working a 40 hour week would complete 40×60 divided by 4 = 600 phone calls per week. With several employees, the CD club surely makes at least 1000 calls per week. At 1000 calls per week, the cost for Apple Communications is \$.95 per call $\times$ 1000 = \$950 per week. The cost for Bell Telephone is \$260 per week + \$.40 $\times$ 1000 calls = \$660 per week. The cost for Capital is only \$600. 1000 calls, as we said, is the minimum. If even more calls were made, then the cost of Bell and the cost of Apple would increase, while the cost of Capital would stay at \$600. Therefore, we'd choose Capital."

Appendix C: Sample Anchor Papers

Anchor papers provided concrete examples to clarify the scoring rubrics. As described in Chapter 3, anchor papers were selected from completed tests used in earlier research conducted by the Core-Plus Mathematics Project. An attempt was made to select anchor papers that defined the borders between scores, particularly in cases where it might be difficult to choose what score to assign. Thus, for example, in addition to

“typical papers with a score of 1,” some anchor papers were selected that were close to “the weakest you could get and still score a 1,” and others were selected that were close to “the strongest you could get and still score a 1.”

This appendix contains sample anchor papers for Part 1, Question 3.3, for Part 2, Question 10, and for Part 3. For readers wishing to use the test and scoring rubric, the full set of anchor papers are available from the author.

Sample Anchor Papers for Part 1, Problem 3.3

∴ updated anchors for 3.3

Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

$$9,112,500 \leq 4500n - 0.2n^2$$

$$n = 2200$$

I put $4500n - 0.2n^2$ in $Y=$. I made a table and when Y was at least 9,112,500 $X = 2200$.

① ①

Question 3.4: For what number of cycle sales will income be as large as possible, and what

00 Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

you need to sell 2925 cycle did
use the table and trial and error

Question 3.4:

0 0 Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

$$9,112,500 = (4500 \cdot n) - (.2)(n^2)$$

\$2110

Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

2064 cycle sales. I entered it into my calculator and looked to where it was flat.

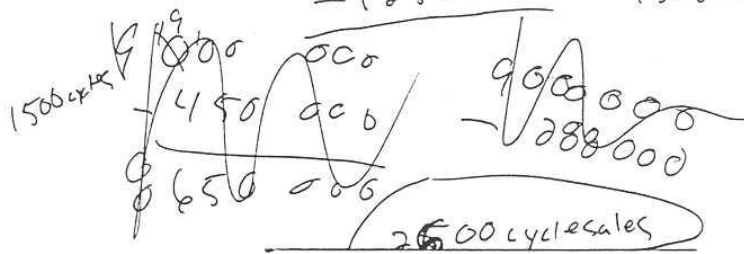
00 Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

$$4506n \approx .2n^2$$

$$\begin{array}{r} 11250000 \\ - 1250000 \end{array}$$

10,340,000

1352000



Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

after approximately 2,105 cycles, the income will be \$9,112,500.

$$(9112500/4500) + .2x^2 = 2,015$$

00

Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

2233

put $4500x - .2x^2$ into $y =$ and look at the table
when y is 9,112,500 or close to it look at
X column which says 2,233 cycles would
have to be sold in order for the income to be
9,112,500

Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

2,203

Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

about
2203

Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

about 2500 cycles

$$C = 800n + 10,050$$

$$I = 4500n - 0.2n^2$$

Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

$$9,112,500 = 4500n - 0.2n^2$$

$$\text{approx} \approx 20247$$

to do this I put the equation
for I into my table and
went until I found my
value of 9,112,500

1.5

$$4500 \cdot 2250 - (1.2) \cdot 2250^2$$

Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

2249
2249

When $n = 2250$,
the income will be
9,112,500.

~~1~~

0.5

Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

2250

~~11~~
0.5

~~11~~ Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

(.5)

The income will be \$9,112,500 for 220 cycle sales

Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

In order to have the income from cycle sales reach @ least \$9,112,500, the cycle sales would have to be 2250.

-5

#

Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

at least 2250 cycles

①

~~25~~
~~45~~

Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

$$I = 4500n - 12n^2$$

I put this in my $y=$ on my calculator
and just changed $I \rightarrow y$ and $n \rightarrow x$

2248, 2249, 2250

$$n \geq 2250$$

~~18~~
~~10~~

①

Sample Anchor Papers for Part 2, Question 10

anchor for "10"

10. Solve the equation $x^2 + 3x - 4 = 0$ for x .

Work Space:

$$\begin{aligned}x^2 + 3x - 4 &= 0 \\x^2 + 3x &= 4 \\3x^2 &= 4 \\x^2 &= 4/3 \\x &= \sqrt{4/3}\end{aligned}$$

Answer:

$$X = \sqrt{4/3}$$

00 10. Solve the equation $x^2 + 3x - 4 = 0$ for x .

Work Space:

$$x^2 - 3x - 4 = 0$$

$$x = 4$$

~~$$5 \cdot 5 = 25 - 15 = 10 - 4 = 6$$~~

~~$$4 \cdot 4 = 16 - 12 = 4 - 4 = 0$$~~

$$4^2 = \underset{12}{16} - 3(4) = 4 - 4 = 0$$

Answer:

$$x = 4$$



10. Solve the equation $x^2 + 3x - 4 = 0$ for x .

Work Space:

$$x^2 + 3x - 4 = 0$$

+4 +4

$$x^2 + 3x = 4$$

$$4x^3 = 4$$

$$x = 1$$

Answer:

$$x = 1$$

student solved
using "fact"
+ let

$$x^2 + 3x = 4x^3$$

Hence, score is "0".

00 10. Solve the equation $x^2 + 3x - 4 = 0$ for x .

Work Space:

Answer:

$x_1 = 1$
 ~~$x_2 = -4$~~

$$\begin{aligned} x^2 + 3x - 4 &= 0 + 4 \\ x^2 + 3x &= 4 \\ 4 \times 3 &= 12 \\ \frac{12}{4} &= 3 \end{aligned}$$

$$\begin{aligned} x^2 + 3x - 4 &= 0 + 4 \\ x^2 + 3x &= 4 \\ \frac{x^2}{4} &= \frac{4}{4} \end{aligned}$$



10. Solve the equation $x^2 + 3x - 4 = 0$ for x .

Work Space:

$$x^2 + 3x - 4 = 0$$

$$x^2 + 3x = 4$$

$$\frac{4x^2}{4} = \frac{4}{4}$$

$$x^2 = 1$$

$$x = 1$$

Answer:

$$x = 1$$

$$x^2 + 3x = 4$$

$$\rightarrow 4x^2 = 4$$

$$\rightarrow x^2 = 1$$

$\rightarrow x = 1$ is
score of "0".



10. Solve the equation $x^2 + 3x - 4 = 0$ for x .

Work Space: $x^2 + 3x - 4 = 0$

$$4x^2 - 4 = 0$$

$$x = 1$$

Answer:

10. Solve the equation $x^2 + 3x - 4 = 0$ for x .

Work Space:

$$x^2 + 3x - 4 = 0$$

$$-1, 4 \quad (x+4)(x-1)$$

Answer:

$$(x+4)(x-1)$$

very high "0"

If student had
written $(x+4)(x-1)$
it would be a "1".

If student had solved
the incorrectly-factored
equation, it would
score a "1".

