

Appendix A: Core-Plus Algebra Test Forms Used In This Study

Part 1: Understanding and Solving Algebra Problems in Context

Reasoning with Algebra and Functions

Part 1

Form C

Student Name _____

Teacher Name _____

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Many problems encountered in business, science, and government require mathematical analysis of relations among several variable quantities. Ideas and methods of algebra and functions are often useful in reasoning about those problems.

The following pages contain 4 problem sets. Each set involves questions about relationships among variables. *For each question write what you believe is the best answer and show the work and reasoning that support your answer.*

If you use a graphing calculator in your work on a question, please explain *briefly* the way you used it (e.g. "I graphed $y = 3x + 2$ and traced looking for $y = 12$ " or "I made a table for $y = x^2$ and looked for x that made $y = 81$.")

You have 40 minutes for the 4 problem sets, so don't spend too long on any one problem or question if you get stuck.

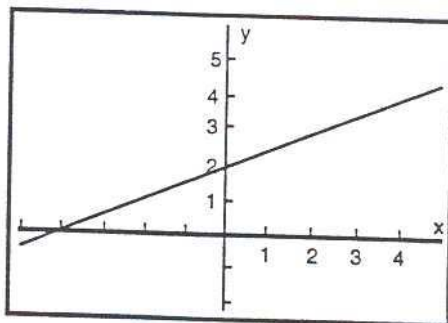
Problem 1: Writing Relationships in Algebraic Form

One of the most important tasks in using methods of algebra to solve problems is finding a way to express problem conditions in symbolic form—function rules, equations, inequalities, etc. That often means translating numerical data, graphs, or written words and sentences into symbolic expressions.

Questions 1.1 - 1.5 give information about relationships between variables and ask you to express that information in equations or function rules.

Question 1.1: The *Watchdog Security Service* provides home and business security systems with installation charge of \$150 and a charge of \$5 per week to monitor the system. Write a formula that gives cost C of the service as a function of time t in weeks that the service is used.

Question 1.2: Write an equation relating x and y that will produce this graph.

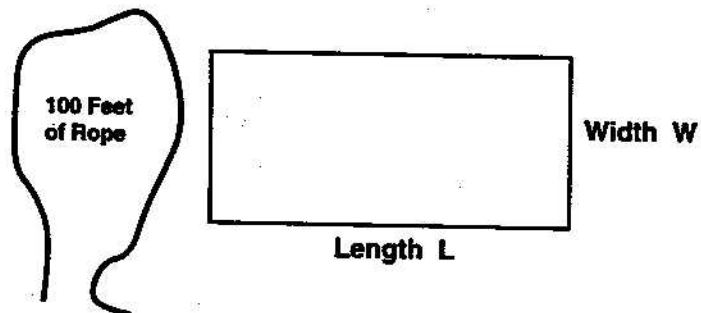


Question 1.3: Pedro and Maria ride a school bus 15 miles to high school. Write an equation that shows how to calculate the average speed S (in miles per hour) of their trip from the time T (in hours) that the trip takes.

Question 1.4: Write an equation relating x and y that will give the pairs of numbers in this table.

x	-1	0	1	2	3
y	0.5	1	2	4	8

Question 1.5: In a school science project each student is given 100 feet of rope to enclose an experimental garden plot in the shape of a rectangle.



- a) Write an equation showing how the *width* W of any such plot depends on the *length* L .

$$W = \underline{\hspace{2cm}}$$

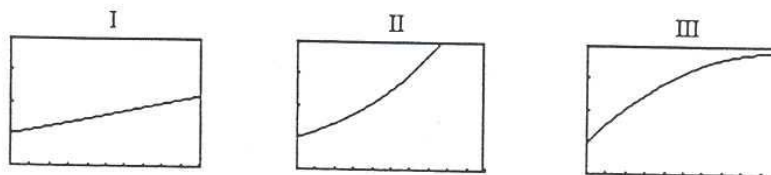
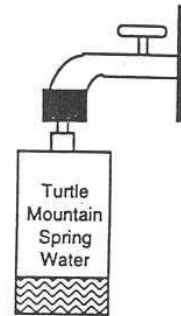
- b) Write another equation showing how the *area* of any plot also depends only on the *length*.

$$A = \underline{\hspace{2cm}}$$

Problem 2: Tapping the Market

Recent reports about bacteria and parasites in city drinking water supplies have led to a booming business in sales of bottled water and water purifying devices. Water has become a big business.

The following graphs show three different predictions of how income from water sales might change over the next several years.



Use the graphs to answer questions 2.1 and 2.2 about water business prospects.

Question 2.1: Which of the three graphs (I, II, or III) shows annual income rising:

- a) at an increasing rate? _____
- b) at a constant rate? _____
- c) at a decreasing rate? _____

Question 2.2: Income from sales of bottled water in 1990 was \$500 million. What is the best match between the three graphs and these equations relating sales income S in millions of dollars to time t in years since 1990?

- a) $S = 500 + 60t$ _____
- b) $S = -15t^2 + 290t + 500$ _____
- c) $S = 500(1.2)^t$ _____

The Turtle Mountain Spring Company made plans about growth in their own share of the water business. They predicted that annual income from sale of their bottled water B and filters F would change over time according to the following formulas. Time t is in years since 1990 and income is in millions of dollars per year.

$$\text{Bottled Water Income: } B = 20 + 5t$$

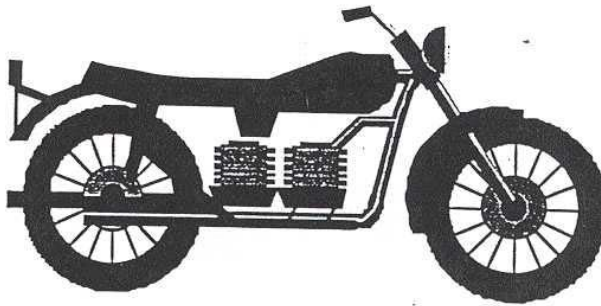
$$\text{Filtering Devices Income: } F = 28 + 3t$$

Use those formulas to answer questions 2.3 - 2.5. Remember to show your work and explain briefly how (if at all) you used a graphing calculator in your work.

Question 2.3: When is the income from bottled water projected to reach \$50 million?

Question 2.4: When does the Turtle Mountain Springs Company expect the two water products to give the same annual income?

Problem 3: The Price Has To Be Right



The builders of a new American Eagle motorcycle plan to set a fixed price for the cycle—no rebates or bargaining allowed. The problem is finding the right price to charge. They estimate that their operating costs c and sales income I will be related to the number n of cycles that are made and sold:

$$\text{Operating Costs: } c = 800n + 10,032,000$$

$$\text{Income from Sales: } I = 4500n - 0.2n^2$$

Answer each of the following questions that arise in studying business prospects for American Eagle's planned new motorcycle. Remember to show your work and explain briefly how (if at all) you used a graphing calculator.

Question 3.1: What will the operating cost be if 5000 cycles are made and sold?

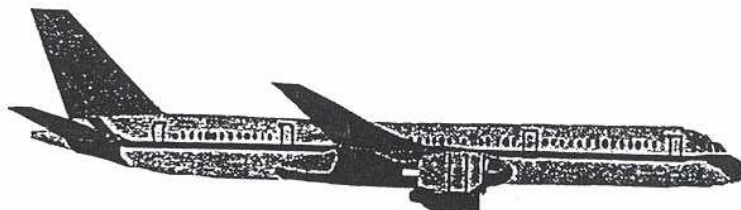
Question 3.2: What will the income from sales be if 2000 cycles are sold?

Question 3.3: For what number of cycle sales will income be at least \$9,112,500?



Question 3.4: For what number of cycle sales will income be as large as possible, and what will that income be?

Problem 4: The Long-Distance Airliner



Several commercial airlines have non-stop flights from Los Angeles, California all the way to Sydney, Australia. It is a trip of 7500 miles and can take as long as 18 hours, most of the time out over the Pacific Ocean. Therefore, estimating flight time and fuel requirements is very important. One airline uses this formula to calculate flight time T in hours from expected wind speed W in miles per hour:

$$T = \frac{7500}{500 + W}$$

Explain briefly what each of the following algebraic calculations will tell about flight plans. You do not need to do the calculations.

Question 4.1: What will you learn from calculating T when $W = 75$?

Question 4.2: What will you learn from solving: $\frac{7500}{500 + W} = 14$?

Question 4.3: What will you learn from solving: $\frac{7500}{500 + W} > 14$?

The *fuel* F in gallons required for a flight is a function of estimated *time* T in hours and number of *passengers* N on the flight. Those variables are related by the equation:

$$F = 2500T + 1.5NT + 1500$$

*Explain briefly what each of the following algebraic calculations will tell about flight plans.
You do not need to do the calculations.*

Question 4.4: What will you learn from calculating F when $T = 14$ and $N = 225$?

Question 4.5: What will you learn by solving $2500T + 1.5NT + 1500 < 48,000$ for N when $T = 16$?

Part 2: Symbol Manipulation

Reasoning with Algebra and Functions

Part 2

Form A

Student Name _____

Teacher Name _____

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On this section of the algebra/functions assessment there are 14 questions. You have 20 minutes to answer them. The questions require algebraic operations that are simple enough to do without use of calculators or computers. **Calculators and computers are not to be used on this section of the assessment.**

Section 2.1

Questions 1-6 are in multiple-choice format. For each question, circle the letter of the choice that you believe is the **best** answer. Use the blank space for scratch work in your reasoning.

1. Which of the following expressions is equivalent to $(4xy^3)(5y)$?

- (a) xy^2
- (b) $9xy^3$
- (c) $9xy^4$
- (d) $20xy^3$
- (e) $20xy^4$

2. Which of the following expressions is equivalent to $\frac{125 + x}{25}$?

- (a) $5x$
- (b) $5 + x$
- (c) $100 + x$
- (d) $\frac{126x}{25}$
- (e) $5 + \frac{x}{25}$

3. Which of the following inequalities is equivalent to $4x - 7 > 3$?

- (a) $x > -1$
- (b) $x > 0$
- (c) $x > \frac{7}{4}$
- (d) $x > \frac{10}{4}$
- (e) $x > 3$

4. If $t \neq 0$, which of the following expressions is equivalent to $\frac{8}{t} - \frac{5}{2t}$?
- (a) $\frac{3}{3t}$
 - (b) $\frac{11}{2t}$
 - (c) $\frac{3}{2t^2}$
 - (d) $\frac{13}{2t^2}$
 - (e) $\frac{3}{-t}$
5. A rectangle has length L and width W . Its length is 4 meters greater than its width. Which of the following describes the relation between L and W ?
- (a) $W = L + 4$
 - (b) $L = W + 4$
 - (c) $W > L + 4$
 - (d) $L + 4 > W$
 - (e) $L > W + 4$
6. Which of the following expressions is equivalent to $-3(5x - 6)$?
- (a) $-15x + 18$
 - (b) $2x - 9$
 - (c) $-15x - 6$
 - (d) $-15x - 18$
 - (e) $2x + 3$

Section 2.2

Give answers for the 8 questions in this section. Use the work space to show your reasoning. Record your results in the "answer" boxes.

7. If $s = 2$ and $t = -3$, then what is $2st + (s^2 t)$?

Work Space:

Answer:

8. Solve the equation $3x + 4 = 5x - 2$ for x .

Work Space:

Answer:

9. Solve the system of equations
$$\begin{cases} -2x + 3y = 8 \\ x - y = 2 \end{cases}$$

Work Space:

Answer:

10. Solve the equation $x^2 + 3x - 4 = 0$ for x .

Work Space:

Answer:

11. Solve $(2x - 6)(x + 4) = 0$ for x .

Work Space:

Answer:

12. Solve $4 = \frac{15}{x+3}$ for x .

Work Space:

Answer:

Part 3: Extended, Applied Algebra Problem

Reasoning with Algebra and Functions

Part 3

Form A

Student Name(s) _____

Teacher Name _____

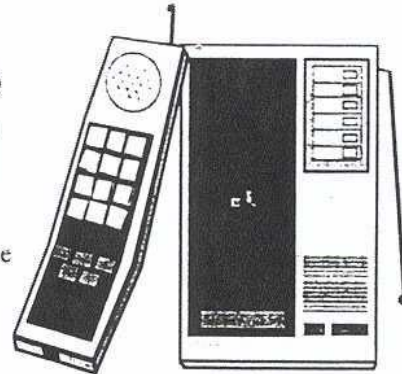
This section of the algebra/functions assessment contains a single problem, based on several pieces of given information. Use the given facts and your mathematical ideas and skills to “solve” the problem.

You have 20 minutes to develop a response working with a partner. You may use a calculator and any other resources that are available for use in your regular homework or classwork assignments. Some graph paper is attached to the question page.

Make notes of ideas and calculations as you work on the problem. *Then combine your ideas to write a clear and complete report of your conclusions, explaining how you arrived at your results, including a brief explanation of how (if at all) you used a graphing calculator.*

Selling by Telephone

Many companies sell their products through long-distance telephone calls to customers. For example, the *CD Club* sells music compact discs all across the country from its headquarters in New York. Sales calls made by the *CD Club* last an average of about 4 minutes apiece. The club has bids from three possible providers for their long-distance telephone service:



- 1) *Apple Communications* will charge \$0.35 for placing each call and then \$0.15 per minute of time used in the call.
- 2) *Bell Telephone* makes a fixed charge of \$260 per week for access to its long-distance lines, but charges only \$0.10 per minute of time used by the calls.
- 3) *Capital Long Distance Services* will make a fixed charge of \$600 per week for unlimited use of its long-distance lines.

Question: For the *CD Club* the problem is to choose the long-distance service that is least expensive for their business. What advice would you give about the phone company to choose and why?

Conclusions and Reasoning:

