

Appendix C: Sample Anchor Papers

Anchor papers provided concrete examples to clarify the scoring rubrics. As described in Chapter 3, anchor papers were selected from completed tests used in earlier research conducted by the Core-Plus Mathematics Project. An attempt was made to select anchor papers that defined the borders between scores, particularly in cases where it might be difficult to choose what score to assign. Thus, for example, in addition to “typical papers with a score of 1,” some anchor papers were selected that were close to “the weakest you could get and still score a 1,” and others were selected that were close to “the strongest you could get and still score a 1.”

This appendix contains sample anchor papers for Part 1, Question 3.3, for Part 2, Question 10, and for Part 3. For readers wishing to use the test and scoring rubric, the full set of anchor papers are available from the author.

Sample Anchor Papers for Part 1, Problem 3.3

∴ updated anchors for 3.3

Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

$$9,112,500 \leq 4500n - 0.2n^2$$

$$n = 2200$$

I put $4500n - 0.2n^2$ in $Y=$. I made a table and when Y was at least 9,112,500 $X = 2200$.

① ①

Question 3.4: For what number of cycle sales will income be as large as possible, and what

00 Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

you need to sell 2925 cycle did
use the table and trial and error

Question 3.4:

0 0 Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

$$9,112,500 = (4500 \cdot n) - (.2)(n^2)$$

\$2110

Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

2064 cycle sales. I entered it into my calculator and looked to where it was flat.

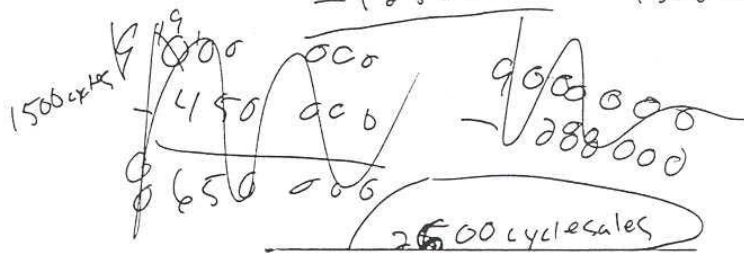
00 Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

$$4506n \approx .2n^2$$

$$\begin{array}{r} 11250000 \\ - 1250000 \end{array}$$

10,340,000

1352000



Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

after approximately 2,105 cycles, the income will be \$9,112,500.

$$(9112500/4500) + .2x^2 = 2,015$$

00

Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

2233

put $4500x - .2x^2$ into $y =$ and look at the table
when y is 9,112,500 or close to it look at
X column which says 2,233 cycles would
have to be sold in order for the income to be
9,112,500

Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

2,203

Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

about
2203

Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

about 2500 cycles

$$C = 800n + 10,050$$

$$I = 4500n - 0.2n^2$$

Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

$$9,112,500 = 4500n - 0.2n^2$$

$$\text{approx} \approx 20247$$

to do this I put the equation
for I into my table and
went until I found my
value of 9,112,500

1.5

$$4500 \cdot 2250 - (1.2) \cdot 2250^2$$

Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

2249
2249

When $n = 2250$,
the income will be
9,112,500.

~~1~~

0.5

Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

2250

~~11~~
0.5

~~11~~ Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

(.5)

The income will be \$9,112,500 for 220 cycle sales

Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

In order to have the income from cycle sales reach @ least \$9,112,500, the cycle sales would have to be 2250.

-5

#

Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

at least 2250 cycles

①

~~25~~
~~45~~

Question 3.3: For what number of cycle sales will income be at least \$9,112,500?

$$I = 4500n - 12n^2$$

I put this in my $y=$ on my calculator
and just changed $I \rightarrow y$ and $n \rightarrow x$

2248, 2249, 2250

$$n \geq 2250$$

~~18~~
~~10~~

①

Sample Anchor Papers for Part 2, Question 10

anchor for "10"

10. Solve the equation $x^2 + 3x - 4 = 0$ for x .

Work Space:

$$\begin{aligned}x^2 + 3x - 4 &= 0 \\x^2 + 3x &= 4 \\3x^2 &= 4 \\x^2 &= 4/3 \\x &= \sqrt{4/3}\end{aligned}$$

Answer:

$$X = \sqrt{4/3}$$

00 10. Solve the equation $x^2 + 3x - 4 = 0$ for x .

Work Space:

$$x^2 - 3x - 4 = 0$$

$$x = 4$$

~~$$5 \cdot 5 = 25 - 15 = 10 - 4 = 6$$~~

~~$$4 \cdot 4 = 16 - 12 = 4 - 4 = 0$$~~

$$4^2 = \underset{12}{16} - 3(4) = 4 - 4 = 0$$

Answer:

$$x = 4$$



10. Solve the equation $x^2 + 3x - 4 = 0$ for x .

Work Space:

$$x^2 + 3x - 4 = 0$$

+4 +4

$$x^2 + 3x = 4$$

$$4x^3 = 4$$

$$x = 1$$

Answer:

$$x = 1$$

student solved
using "fact"
+ let

$$x^2 + 3x = 4x^3$$

Hence, score is "0".

00 10. Solve the equation $x^2 + 3x - 4 = 0$ for x .

Work Space:

Answer:

$x_1 = -4$
 $x_2 = 1$

$$\begin{aligned} x^2 + 3x - 4 &= 0 + 4 \\ x^2 + 3x &= 4 \\ 4 \times 3 &= 12 \\ \frac{12}{4} &= 3 \end{aligned}$$

$$\begin{aligned} x^2 + 3x - 4 &= 0 + 4 \\ x^2 + 3x &= 4 \\ \frac{x^2}{4} &= \frac{4}{4} \end{aligned}$$



10. Solve the equation $x^2 + 3x - 4 = 0$ for x .

Work Space:

$$x^2 + 3x - 4 = 0$$

$$x^2 + 3x = 4$$

$$\frac{4x^2}{4} = \frac{4}{4}$$

$$x^2 = 1$$

$$x = 1$$

Answer:

$$x = 1.$$

$$x^2 + 3x = 4$$

$$\rightarrow 4x^2 = 4$$

$$\rightarrow x^2 = 1$$

$\rightarrow x = 1$ is
score of "0".



10. Solve the equation $x^2 + 3x - 4 = 0$ for x .

Work Space: $x^2 + 3x - 4 = 0$

$$4x^2 - 4 = 0$$

$$x = 1$$

Answer:

10. Solve the equation $x^2 + 3x - 4 = 0$ for x .

Work Space:

$$x^2 + 3x - 4 = 0$$

$$-1, 4 \quad (x+4)(x-1)$$

Answer:

$$(x+4)(x-1)$$

very high "0"

If student had
written $(x+4)(x-1)$
it would be a "1".

If student had solved
the incorrectly-factored
equation, it would
score a "1".

10. Solve the equation $x^2 + 3x - 4 = 0$ for x .

11 ~~X~~

Work Space:

$$1^2 + 3 \times 1 - 4 = 0$$

Answer:

/

substituting

"1" or "-4"

in the equation
to show that it
works is a "1".

10. Solve the equation $x^2 + 3x - 4 = 0$ for x .

Work Space:

Answer:

$$x = 1$$

note work over
the equation:

correct substitution
for "guess and check"
answer.

10. Solve the equation $x^2 + 3x - 4 = 0$ for x .

Work Space:

$$x^2 + 3x - 4 = 0$$

$$1^2 + 3(1) - 4 = 0$$

$$1 + 3 - 4 = 0$$

Answer:	$1^2 + 3(1) - 4 = 0$
$x = 1$	

10. Solve the equation $x^2 + 3x - 4 = 0$ for x .

Work Space:

$$x^2 + 3x - 4 = 0$$

$$(x + 4)(x - 1)$$

$$x^2 - 1x + 4x - 3$$

Answer:

$$(x + 4)(x - 1)$$

10. Solve the equation $x^2 + 3x - 4 = 0$ for x .

Work Space:

$$(x+4)(x-1)$$

$$x^2 - 1x + 4x - 4$$

Answer:

$$x = -4, 1$$

$$16 + 12 - 4$$

very high "1")

10. Solve the equation $x^2 + 3x - 4 = 0$ for x .

Work Space:

Answer:

$$x = 4, -1$$

$$\begin{aligned}x^2 + 3x - 4 &= 0 \\(x + 4)(x - 1) &= 0\end{aligned}$$

- 22 10. Solve the equation $x^2 + 3x - 4 = 0$ for x .

Work Space:

$$\begin{aligned}x^2 + 3x - 4 &= 0 \\(x + 4)(x - 1) &= 0 \\x &= -4 \text{ or } 1\end{aligned}$$

Answer:

$$x = 4 \text{ or } -1$$

- 22 11. Solve $(2x - 6)(x + 4) = 0$ for x .

Work Space:

$$(2x - 6)(x + 4) = 0$$

Answer:

$$x = 3 \text{ or } 4$$

33 10. Solve the equation $x^2 + 3x - 4 = 0$ for x .

Work Space:

$$x^2 + 3x - 4 = 0$$

$$(x - 1)(x + 4)$$

$$\begin{array}{r} x - 1 = 0 \\ +1 \quad +1 \\ \hline x = 1 \end{array}$$

$$\begin{array}{r} x + 4 = 0 \\ -x \quad -4 \\ \hline x = -4 \end{array}$$

Answer:

$$x = 1, -4$$

Sample Anchor Papers for Part 3

This section contains 8 of 35 anchor papers used for Part 3. The full set of anchor papers can be ordered from the author.

Selling by Telephone

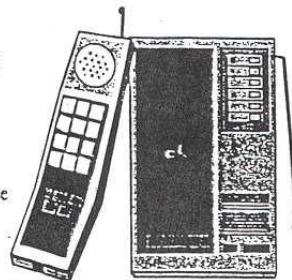
correct underlying model ✓

correct explanation ✓

(although it would be better to explain why the "intersections" helped decide which was the correct company)

correct answer ✓

ducts through long-
For example, the CD
ross the country from
alls made by the CD
s apiece. The club
for their long-distance



will charge \$0.35 for placing each call and then \$0.15 per minute in the call.

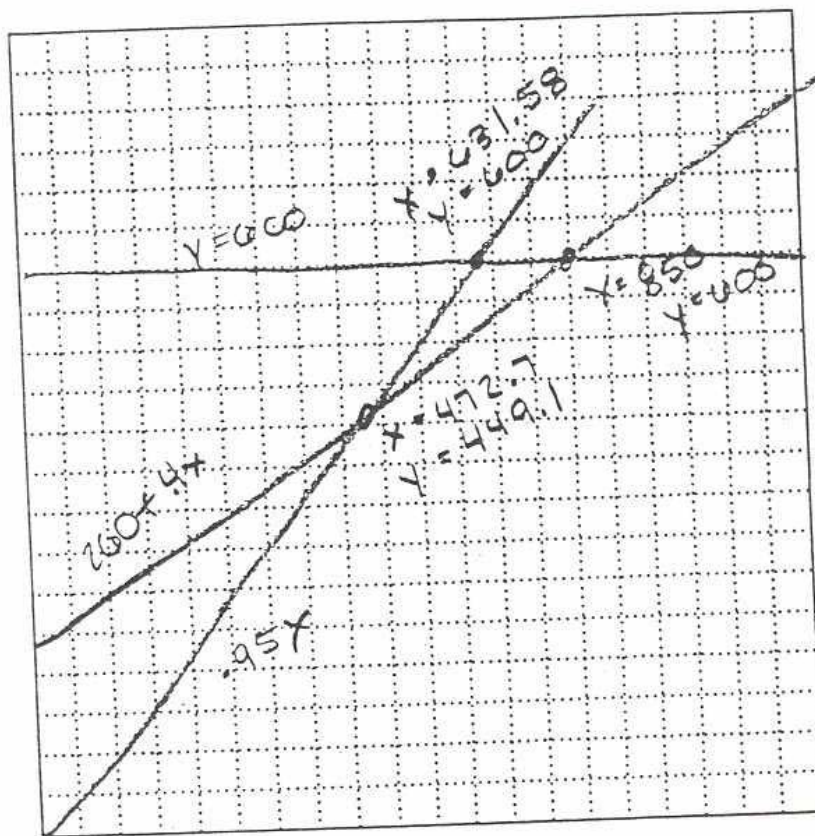
- 2) Bell Telephone makes a fixed charge of \$260 per week for access to its long-distance lines, but charges only \$0.10 per minute of time used by the calls.
- 3) Capital Long Distance Services will make a fixed charge of \$600 per week for unlimited use of its long-distance lines.

Question: For the CD Club the problem is to choose the long-distance service that is least expensive for their business. What advice would you give about the phone company to choose and why?

Conclusions and Reasoning:

$$\begin{aligned}
 \text{Apple} &= .15T + .35 \quad \text{Apple } 195 \times / \text{call} \quad 95 \times \\
 \text{Bell} &= .1T + 260 \quad 404 / \text{call} + 260 / \text{week} \quad 260 + 4x \\
 (\text{qp.} &= 600 \quad \text{unlimited @ } 600 \quad y = 600
 \end{aligned}$$

(continued on next page)



we formed "y=" equations for a 4min. telephone call and then graphed them. (A sketch is drawn above). We used the intersections to decide the best companies. Apple communications is the best company if they make less than 472 calls per week. Bell telephone offers the best deal if they plan on making between 473 and 850 calls per week. If the plan on making over 850 calls per week then Capital Long Distance Services offers the best deal.

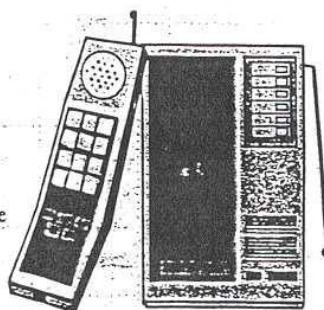
Anchor Paper: ^{score} 3

Correct model ✓
explanation inadequate ☹️
(A graph alone is
not enough.)

correct answer ✓
note: rounding is
accepted.

by Telephone

through long-
example, the CD
country from
made by the CD
ece. The club
their long-distance



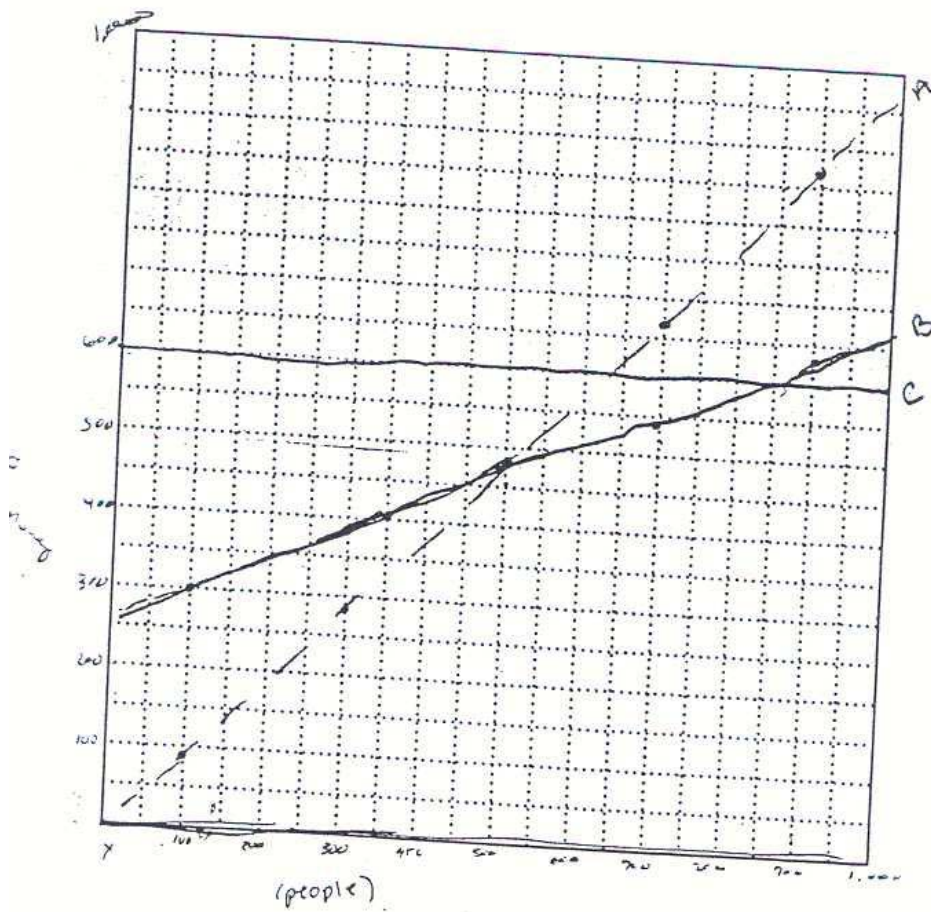
- 1) Apple Communications will charge \$0.35 for placing each call and then \$0.15 per minute of time used in the call. $y = 0.35 + 0.15x$
- 2) Bell Telephone makes a fixed charge of \$260 per week for access to its long-distance lines, but charges only \$0.10 per minute of time used by the calls. $y = 260 + 0.10x$
- 3) Capital Long Distance Services will make a fixed charge of \$600 per week for unlimited use of its long-distance lines. $y = 600$

Question: For the CD Club the problem is to choose the long-distance service that is least expensive for their business. What advice would you give about the phone company to choose and why?

Conclusions and Reasoning:

calling less than 500 people use Apple
calling between 500 and 850 use Bell
calling more than 850 use Capital

(continued on next page)



500 people
or less A
500 to 850 people
B
850 or
more
people
C

.95P
260 + .4P ...
600

anchor paper (3)

correct model ✓

(a careful reading indicates both costs for Bell and for Apple are modeled correctly)

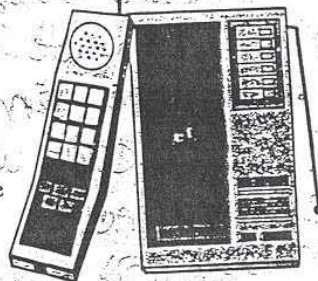
correct explanation ✓

(you can follow their logic)

answer incorrect (in)

(they did not consider case where Apple might be cheaper than Bell, even though Bell is cheaper than Capital)

phone



or placing each call and then \$0.15 per

\$0 per week for access to its long-minute of time used by the calls.

fixed charge of \$600 per week for

Question: For the CD Club the problem is to choose the long-distance service that is least expensive for their business. What advice would you give about the phone company to choose and why?

Conclusions and Reasoning:

$$600 - 260 = 340$$

$$340 / 4 = 850$$

$$3400 / 4 = 850$$

~~conclusion~~ can order for the bell telephone company's price to exceed capital long distance service's price. But the CD club would have to spend over 3400 minutes per week on the phone. we don't think they will be spending this much time b/c (averagely 4 minutes a call) they would have to make 850 calls. ^{page 2} saying hypo-

Algebra Assessment - Part 3 Form A

the CD club does make 850

(continued on next page)

calls per week then w/ Apple Communications they'd have to pay \$.35 for each call + \$.15 for each minute of the call. This equals \$.45 for a 4 minute call; therefore 850 4 minute calls would cost \$801.50.

We can conclude that if the CO club makes 850 calls (or less) per week Bell Telephone will give them the best deal, but if they make more than 850 calls per week Capital Long Distance Services will give them the best deal.

anchor paper: strong "2"

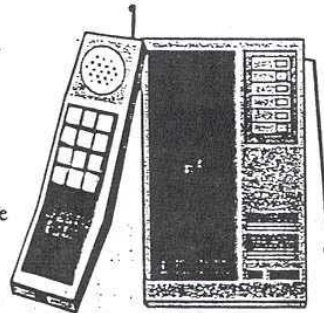
incorrect model (n)

inadequate explanation (n)

But note: had they drawn
their graph accurately, and discussed
the point where Bell cost = Apple Cost
at calls = 0, this could have
been a "3"
answer is consistent with model ✓
telephone service:

ling by Telephone

ducts through long-
For example, the CD
cross the country from
calls made by the CD
ites apiece. The club
s for their long-distance



- 1) Apple Communications will charge \$0.35 for placing each call and then \$0.15 per minute of time used in the call.
- 2) Bell Telephone makes a fixed charge of \$260 per week for access to its long-distance lines, but charges only \$0.10 per minute of time used by the calls.
- 3) Capital Long Distance Services will make a fixed charge of \$600 per week for unlimited use of its long-distance lines.

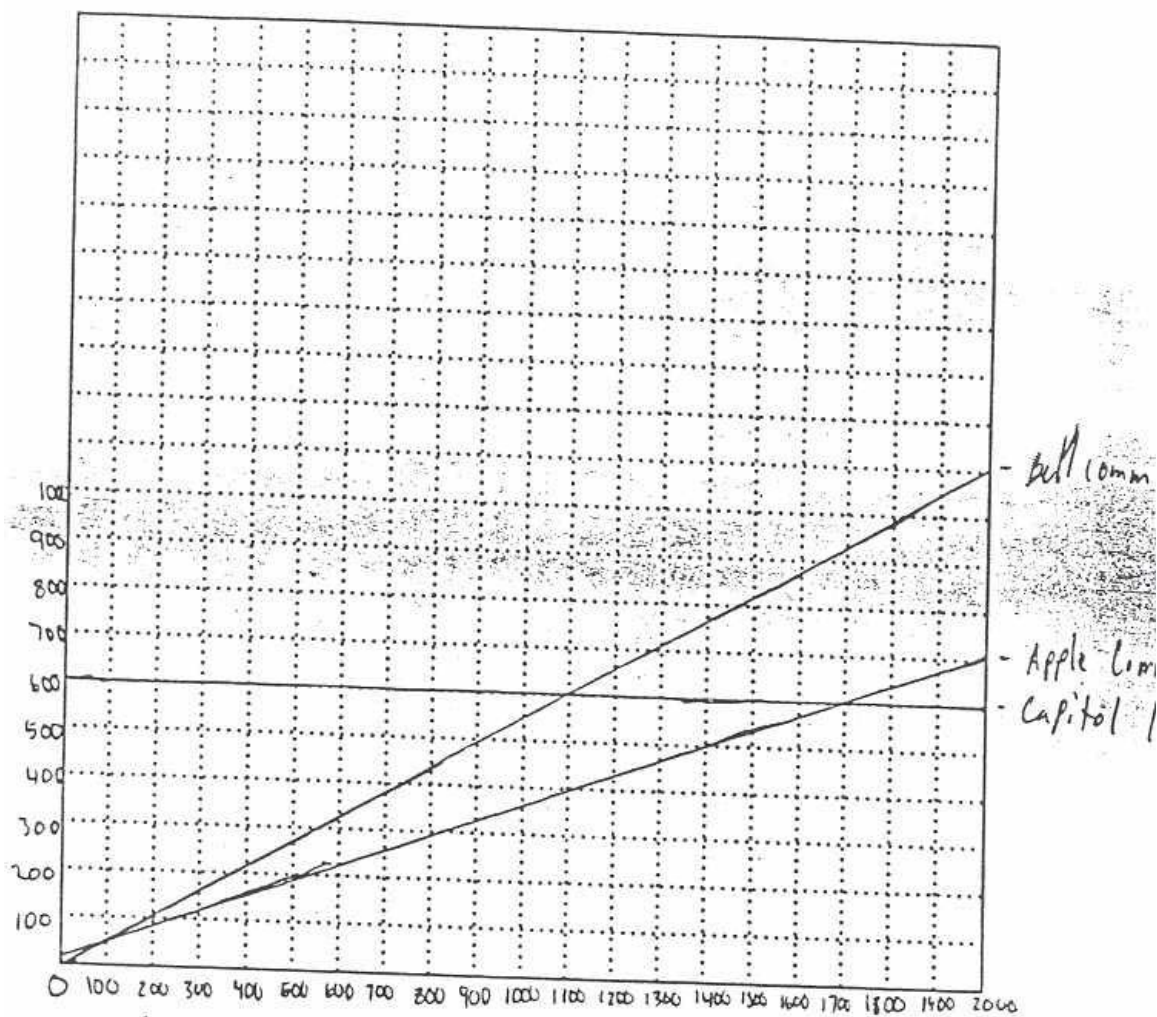
Question: For the CD Club the problem is to choose the long-distance service that is least expensive for their business. What advice would you give about the phone company to choose and why?

Conclusions and Reasoning:

When making 0 to 1,712 calls per week use Apple communications, when making 1,713 or more calls per week use Capital long distance services to get the cheapest rates.

- From the information given about the 3 phone services rates we made an equation for each & made a table & graph for each to see which was cheapest

(continued on next page)



Anchor Paper "2"

- correct model ✓

(see .95 a call note
near top of page)

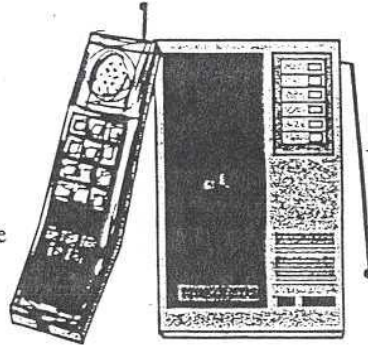
- explanation missing

- answer incorrect (i)
(discrete case)

telephone service:

ing by Telephone

ducts through long-
For example, the CD
cross the country from
calls made by the CD
tes apiece. The club
s for their long-distance



.95 a call that last 4 min. 4.75

- 1) Apple Communications will charge \$0.35 for placing each call and then \$0.15 per minute of time used in the call.
- 2) Bell Telephone makes a fixed charge of \$260 per week for access to its long-distance lines, but charges only \$0.10 per minute of time used by the calls.
- 3) Capital Long Distance Services will make a fixed charge of \$600 per week for unlimited use of its long-distance lines.

Question: For the CD Club the problem is to choose the long-distance service that is least expensive for their business. What advice would you give about the phone company to choose and why?

Conclusions and Reasoning:

I would choose plan 2. The fixed charge of \$260 per week for access to its long distance lines, and charge \$0.10 per min. this plan seems cheap only talking 4 min. at a time. is .40¢ a phone call plus the \$260 fixed charge.

Anchor "1"

Discrete case

model (n) discrete

explanation (n) close,

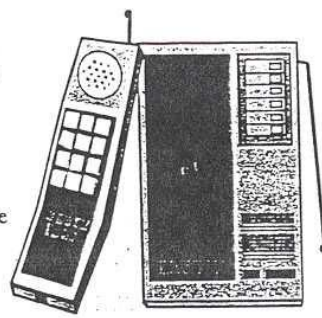
but choice of "500"

not adequately justified

answer (n) only 1 value considered.

by Telephone

through long-
example, the CD
he country from
made by the CD
piece. The club
their long-distance



707
764

telephone service:
 $.10 \times 4 \times 500 = 200$

500 calls/week

$500 \times .35 = 175$

$.15 \times 4 \times 500 = 300$

- 1) Apple Communications will charge \$0.35 for placing each call and then \$0.15 per minute of time used in the call. $175 + 300 = \$475$
- 2) Bell Telephone makes a fixed charge of \$260 per week for access to its long distance lines, but charges only \$0.10 per minute of time used by the calls. $260 + 200 = \$460$
- 3) Capital Long Distance Services will make a fixed charge of \$600 per week for unlimited use of its long-distance lines. 600

Question: For the CD Club the problem is to choose the long-distance service that is least expensive for their business. What advice would you give about the phone company to choose and why?

Conclusions and Reasoning:

WE ESTIMATED THE COMPANY WOULD MAKE 500 CALLS A WEEK. THEN WE WENT THROUGHT TO EACH COMPANY TO FIND OUT HOW MUCH IT WOULD COST FOR 500 CALLS A WEEK. (A) CAME OUT TO \$475 (B) WAS \$460 AND (C) WAS \$600. IF THE COMPANY WERE TO MAKE AN AVERAGE OF 500 CALLS PER WEEK, THE CHEAPEST WOULD BE BELL TELEPHONE.

anchor paper "1"

377
462

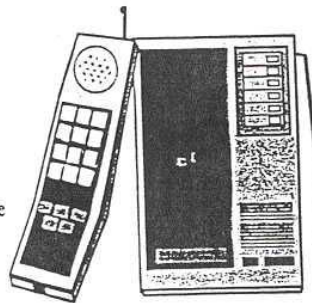
incorrect model (i)

inadequate explanation (i)

muddled answer (i)
not clearly consistent
with model

elling by Telephone

products through long-
ers. For example, the CD
l across the country from
les calls made by the CD
minutes apiece. The club
ders for their long-distance



telephone service.

- 1) Apple Communications will charge \$0.35 for placing each call and then \$0.15 per minute of time used in the call.
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- 3) Capital Long Distance Services will make a fixed charge of \$600 per week for unlimited use of its long-distance lines.

Question: For the CD Club the problem is to choose the long-distance service that is least expensive for their business. What advice would you give about the phone company to choose and why?

Conclusions and Reasoning:

$$\begin{aligned} 1) y &= .35 + .15x - 3997 \text{ min.} \\ 2) y &= 260 + .1x - \text{before } 3400 \text{ min.} \\ 3) y &= 600 = \text{after } 3400 \text{ min. (2)} \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} 5193$$

If the CD club wants to use the fewest minutes they would use plan #1 after 3997 minutes they would want to use plan #3. Plan #3 is the best unless you

UPDATED (1st page updated)

Key for Part I, Problem 3 Practice A

Problem 3.1

A ₁	0
A ₂	1
A ₃	2
A ₄	2
A ₅	2

Problem 3.2

A ₁	1
A ₂	2
A ₃	2
A ₄	0
A ₅	0

~~IF discussion of 3.1
assigns a score
of ".5" then A₄
should also be ".5" not "2"~~

~~IF discussion of 3.2
assigns 8.2 E6E 82,000,000
a score of "1", then
A₂ should also be
a "1", not a "2"~~

Problem 3.3

A ₁	.5
A ₂	.5
A ₃	1.0
A ₄	.5
A ₅	0

Problem 3.4

A ₁	1
A ₂	0
A ₃	2
A ₄	2
A ₅	1