

Appendix E: Detailed Description of IMP Modules

This appendix contains a parent handout prepared by Suburban High School mathematics teachers. The handout lists the IMP modules contained within each IMP text, and contains a detailed description of mathematics concepts and skills encountered in each module. Information in the handout was developed from Teacher's Guides for each IMP module, which are published by Key Curriculum Press.

THE INTEGRATED MATH PROGRAM

INTEGRATED MATH 1 TEXT

Patterns
The Game of Pig
The Overland Trail
The Pit and the Pendulum
Shadows

INTEGRATED MATH 2 TEXT

Solve It!
Is There Really a Difference?
Do Bees Build It Best?
Cookies
All About Alice

INTEGRATED MATH 3 TEXT

Fireworks
Orchard Hideout
Meadows or Malls?
Small World, Isn't It ?
Pennant Fever

INTEGRATED MATH 4 TEXT

High Dive
World of Functions
Know How
As the Cube Turns
Pollster's Dilemma

Integrated Math 1 Content

The course is divided into 5 units. The following is a summary of the concepts and skills students will encounter and practice in each unit.

1. PATTERNS

Students will:

- * analyze and create In-Out tables
- * use variables to express generalizations
- * find, analyze, and generalize patterns
- * work with order of operations for arithmetic
- * use a concrete model to understand and do arithmetic with positive and negative numbers
- * work with geometric concepts
- * write a calculator program to simulate an In-Out machine
- * develop proofs

2. THE GAME OF PIG

Students will:

- * develop and analyze strategies
- * express probability as a number between zero and one
- * calculate probabilities based on equally likely events and area models
- * decide whether events are independent
- * use "the long run" to develop the concept of expected value
- * calculate and interpret expected values
- * solve problems involving conditional probability
- * construct mathematical models for probabilistic situations
- * make and interpret frequency bar graphs
- * use simulations to estimate probabilities
- * compare the theoretical analysis of a situation with experimental results
- * write a calculator program to create a simulation
- * use a computer simulation to compare strategies

3. THE OVERLAND TRAIL

Students will:

- * create examples that fit a set of constraints
- * interpret ambiguous problems
- * find numbers that fit several conditions
- * compile and organize data
- * use data to make decisions
- * develop numerical algorithms
- * express algorithms in words and symbols
- * develop meaningful algebraic expressions
- * work with subscript notation

- * interpret graphs intuitively
- * use graphs to represent equations
- * understand the connections between situations, graphs, In-Out tables, and algebraic rules
- * make graphs from tabular information
- * find lines of best fit intuitively
- * use tables of information and lines of best fit to make predictions
- * graph and use zoom and trace facilities on a graphing calculator
- * solve equations for one variable in terms of others
- * combine two linear conditions
- * use the point of intersection of graphs to satisfy two conditions
- * work with rate problems

4. THE PIT AND THE PENDULUM

Students will:

- * perform scientific experiments
- * plan controlled experiments
- * formulate and refine conjectures
- * collect and analyze data
- * express results using bar graphs
- * calculate mean and standard deviation
- * analyze graphs of quadratic equations using computer software
- * use calculators to explore curve fitting
- * express mathematical ideas orally and in writing

5. SHADOWS

Students will:

- * plan controlled experiments
- * work in groups performing scientific experiments
- * formulate and refine conjectures
- * collect and analyze data
- * use calculators to explore curve fitting
- * discover criteria for polygons to be similar
- * rediscover the angle sum properties of polygons
- * discover the triangle inequality relationship and extend it to polygons
- * acquire intuition about angles by working with protractors
- * learn informal procedures for solving proportions
- * connect geometric pictures with proportions
- * apply properties of similar triangles to physical situations
- * learn the definitions of sine, cosine, and tangent to solve problems
- * express mathematical ideas orally and in writing

Integrated Math 2 Content

The course is divided into 5 units. The following is a summary of the concepts and skills students will encounter and practice in each unit.

1. SOLVE IT

Students will:

- develop equations to represent problem situations
- review substitution and order of operations
- understand the relationship between an algebraic rule, a table, and a graph
- review function notation
- recognize equations leading to straight line graphs
- represent real-life situations using functions
- create equivalent equations
- understand and use the distributive property
- identify and solve linear equations
- investigate how the parts of a linear function affect its graph
- define functions using expressions from equations
- use graphing calculators to solve a variety of equations

2. IS THERE REALLY A DIFFERENCE?

Students will:

- construct, and draw inferences from charts, tables, and graphs that summarize data from real-world situations
- understand that data snooping and hypothesis testing are two different activities
- design a statistical experiment to study a problem, conduct the experiment, and interpret and communicate the outcomes
- make hypotheses about larger populations by looking at sample data
- make null hypotheses to help evaluate the validity of and clarify hypotheses
- develop intuition about when differences in samples mean it is likely that the larger populations are different
- work with conditional probabilities
- calculate and interpret a chi-square statistic for given data to test if it fits a theoretical model
- calculate a chi-square statistic for given data about two populations
- use proportional reasoning to analyze the consequences of null hypothesis
- understand why chi-square statistics are calculated using a square and a proportion
- understand how a probability chart is associated with the chi-square statistic and what the probabilities mean
- use chi-square statistics to make decisions
- understand some limitations in applying a chi-square statistic to a situation
- express mathematical ideas orally and in writing

3. DO BEES BUILD IT BEST?

Students will:

- * gain an intuitive understanding of the concepts of area of 2- dimensional figures and surface area and volume of 3- dimensional figures
- * measure the area and volume of irregular shapes
- * discover formulas for area of triangles, rectangles, trapezoids, and regular polygons
- * write proofs
- * review triangle inequality
- * apply the Pythagorean Theorem
- * work with irrational numbers
- * create formulas to express generalizations in geometric situations
- * find the maximum of a function using a graphing calculator
- * review trigonometric functions
- * discover relationships that exist between perimeter and area of circles and regular polygons
- * discover formulas for the volume and area of prisms
- * understand that for a prism, maximizing volume for a fixed lateral surface area means maximizing area of the base
- * discover criteria for regular polygons to tessellate

4. COOKIES

Students will:

- express and interpret constraints using inequalities
- graph systems of inequalities
- see that setting a linear expression equal to different constants gives a family of parallel lines
- find the maximum of a linear function over a region
- realize that a system of two linear equations in two unknowns can have no solutions, one solution, or an infinite number of solutions
- recognize that different equations can have the same graph
- relate the idea of intersection of graphs to the idea of common solution of equations
- solve linear equations for one variable in terms of another
- use graphing calculators to draw feasible regions
- use graphing calculators to estimate points of intersection of graphs
- examine how the parameters in a problem can affect the nature of the solutions
- discover and describe the method of substitution for solving two equations
- create and solve problems by setting up and solving two equations
- make clear and convincing arguments
- recognize problems with inconsistent conditions
- study, and develop conclusions about, linear combinations
- create and solve linear programming problems with two variables

5. ALL ABOUT ALICE

Students will:

- extend the concept of exponent to include zero, negative, and fractional exponents
- develop and use the Additive Law of Exponents, $a^b \times a^c = a^{b+c}$, and other general properties of exponents
- develop and analyze the graph of the function $y = 2^x$ and related functions
- define and work with logarithms, both on paper and using calculators
- write numbers using scientific notation and use this notation to simplify computation
- work with scientific notation on calculators
- learn about order of magnitude, and use this concept in estimation
- understand the concept of significant digits, and use this concept in context
- work with different units of measurement, and convert from one to another
- be introduced to and use standard $f(x)$ function notation
- study and use basic ideas of logical inference
- work with mirror symmetry

The course is divided into 5 units. The following is a summary of the concepts and skills students will encounter and practice in each unit.

1. FIREWORKS

Students will:

- use formulas in solving problems
- learn terminology related to quadratic expressions, functions, and equations
- understand the role of the vertex and the x-intercept in the graphs of quadratic functions
- recognize the significance of the sign of the X^2
- transform quadratic expressions into vertex form
- apply graphing to solving problems involving quadratics
- use the zero property of multiplication to solve quadratics
- use factoring to solve quadratics
- relate factoring of quadratics to problems of area
- understand meaning of "not factorable"
- identify certain expressions as perfect squares

2. ORCHARD HIDEOUT

Students will:

- use the Cartesian coordinate system to organize a complex problem
- review the concepts of area and the Pythagorean Theorem
- find the equation for a circle with an arbitrary center and radius
- understand the use of the phrases "if . . . , then. . ." and "if and only if" in definitions and proofs
- define the concept of congruence and examine its relationship to similarity
- discover and prove that the set of points equidistant from two given points is the perpendicular bisector
- prove that the perpendicular bisectors of a triangle must meet in a common point
- find criteria which will guarantee the triangles are congruent
- find the relationship of the circumference and area of a circle to its radius
- discover that the ratios of areas and circumferences of circles to areas and perimeters of circumscribed regular polygons are independent of the radius of the circles
- develop formulas for the area and perimeter of polygons circumscribed about a circle, and use these to develop and understand formulas for the area and circumference of a circle
- learn about the significance of pi
- discover that any line through the midpoint of a segment is equidistant from the endpoints
- discover and prove that the set of points equidistant from two intersecting lines is the union of the bisectors of the angles formed by the lines
- prove that the three angle bisectors of a triangle must meet in a common point
- find ruler and straightedge constructions for various geometric configurations
- develop and apply the distance and midpoint formulas
- practice spatial visualization
- use a compass to create art

3. Meadows or Malls?

Students will:

- Express constraints stated in words as inequalities using algebra
- Find numbers that satisfy constraints
- Review linear programming with two variables
- Solve systems of more than two equations by substitution
- Solve linear systems of equations by elimination
- Geometrically represent systems of linear equations in two and three variables
- Study the possible ways in which sets of lines and planes intersect
- Use algebra to write a proof
- Develop definitions for addition and multiplication of matrices
- Express complex manipulations in words
- Learn which algebraic laws hold for operations on the real numbers
- Learn which algebraic laws hold for matrix operations
- Learn that matrix equations are equivalent to systems of linear equations
- Understand the use of matrix algebra to solve systems of linear equations
- Find matrix inverses on paper and with a TI-82/83
- Understand the relationship of inconsistent and dependent equations to non invertible matrices
- Use the TI-82/83 and inverses of matrices to solve systems of linear equations
- Present the analysis and solution of complex linear programming problems

4. SMALL WORLD

Students will:

- analyze rates of change from graphs
- realize that, other things being equal, the rate of change of population growth is proportional to the population
- discover and prove that lines have a constant slope and rate of change
- understand the relationship between the rate of change of a function and the appearance of its graph
- develop an algebraic definition of slope
- understand the significance of a negative slope for the appearance of a graph
- develop the concept of the derivative of a function at a point as a generalization of the notion of slope
- numerically estimate derivatives of curves at points
- strengthen understanding of logarithms
- discover and explain the properties and relationships of logarithms and exponents
- realize the derivative of exponential functions are proportional to their values
- characterize situations appropriately with exponential functions
- develop recursive rules
- estimate the value for e as the base of the exponential function whose derivative equals its value
- develop and use a formula for compound interest
- learn the meaning of the terms natural and common logarithms
- use exponential functions to fit data
- explore patterns and find formulas concerning arithmetic sequences and their partial sums

5. Pennant Fever

Concepts and Skills

Here is a summary of the main concepts and skills that students will encounter and practice in this unit.

Probability and statistics

- Developing a mathematical model for a complex probability situation
- Using area diagrams and tree diagrams to find and explain probabilities
- Using a simulation to understand a situation, to help analyze probabilities, and to support a theoretical analysis
- Finding expected value
- Finding and using probabilities for sequences of events
- Using specific problem contexts to develop the binomial distribution, and finding a formula for the associated probabilities
- Using probability to evaluate null hypotheses

Counting principles

- Developing systematic lists for complex situations
- Using the multiplication principle for choosing one element from each of several sets
- Defining and using the concepts of permutation and combination
- Understanding and using standard notation for counting permutations and combinations
- Developing formulas for the permutation and combinatorial coefficients

Pascal's triangle and combinatorial coefficients

- Finding patterns and properties within Pascal's triangle
- Recognizing that Pascal's triangle consists of combinatorial coefficients
- Explaining the defining pattern and other properties of Pascal's triangle using the meaning of combinatorial coefficients
- Developing and explaining the binomial theorem

Other concepts and skills are developed in connection with Problems of the Week.

Integrated Math 4 Content

The course is divided into 5 units. The following is a summary of the concepts and skills students will encounter and practice in each unit.

1. High Dive

Students will:

- Review basics about circles, angles, and right triangle trigonometry,
- Review the concept of periodicity,
- Examine a complex problem to determine what information is needed to solve it,
- Find a quadrant-by-quadrant expression for circular motion.
- Extend the sine and cosine functions to all angles, based on both physical models and coordinate reasoning,
- See the importance of similarity in the definition of the sine and cosine functions,
- Connect the sine and cosine functions to Ferris wheel problems.
- Graph the sine and cosine functions,
- Define the inverse sine function and principal values,
- Work with varying parameters to model examples of periodicity,
- Review the concept of instantaneous velocity and its estimation,
- Review the derivative and its relationship to instantaneous velocity,
- Examine the relationship between average to instantaneous velocity,
- Understand why, for situations with constant acceleration, the average velocity for any interval is equal to the average of the instantaneous velocities of the two endpoints of the interval,
- Develop quadratic expressions for the height of free-falling objects, based on the physical principle of constant acceleration,
- Study the relationship among acceleration, velocity, and speed, with special attention to the signs of acceleration and velocity,
- See the importance of quadratic equations in studying falling objects
- learn how to use the quadratic formula to solve quadratic equations
- discover and explain trigonometric identities
- define polar coordinates and develop expressions for rectangular coordinates in terms of polar coordinates
- express velocity in terms of its vertical and horizontal components
- study the motion of falling objects and find a general solution for the falling time of objects with an initial vertical velocity

2. The World of Functions

Students will:

- review different ways to think about functions
- formally define functions as sets of ordered pairs
- sketch graphs of functions based on situations
- review basic families of functions
- find, describe, and prove patterns in the tables of functions based on the algebraic form
- study the concept of direct and inverse proportionality and their constants
- find vertical and horizontal asymptotes for specific functions and vice versa
- find patterns in the graphs of power functions
- compare models of linear and exponential growth
- develop a measure of "quality of fit" for a function to a set of data
- compare functions using the least squares approximation
- use calculator regression to find a function that fits the data
- use absolute value and step functions to model problem situations
- describe situations using arithmetic combinations of functions
- formally define arithmetic operations on functions
- relate arithmetic operations on functions to graphs
- use rational functions to model problem situations
- understand situations involving composition of functions
- define composition and composition notation
- understand situations involving inverse functions
- formally define inverse functions
- relate the concept of inverse function to graphs and tables
- find the inverse of specific functions
- work with iteration of functions

3. Know How

Students will:

- use factoring to solve equations
- learn and use radian measure
- prove the quadratic formula
- use the Laws of Sines and Cosines
- perform complex number operations
- study the ellipse

4. As The Cube Turns

Students will:

- learn to use a technical manual
- use loops in programming
- express geometric transformations in terms of coordinates in two and three dimensions
- review the algebra of matrices
- use matrices to express geometric transformations
- derive the formula for the area of a triangle in terms of the sine of an angle and adjacent sides
- derive formulas for the sine and cosine of the sum of two angles
- understand programs from their listings
- find the projection of a point on a plane from the perspective of a fixed point, and develop an algebraic description of the projection process
- study the effect of change of viewpoint on projections
- use spatial reasoning in two and three dimensions

5. Pollster's Dilemma

Concepts and Skills

Here is a summary of the main concepts and skills that students will encounter and practice in this unit.

General concepts for sampling

- Establishing methods of good polling, including random sampling
- Using sampling from a known population to analyze reliability of samples
- Distinguishing between "sampling with replacement" and "sampling without replacement," and comparing the two methods
- Using the terminology of "true proportion" and "sample proportion"
- Identifying simplifying assumptions in analyzing sampling

Specific results on sampling with replacement

- Making probability bar graphs for various distributions
- Developing the concept of a theoretical distribution for sampling results from a given population
- Using combinatorial coefficients to find the theoretical distribution of poll results for polls of various sizes
- Generalizing that sampling results fit a binomial distribution

The central limit theorem and the normal distribution

- Seeing intuitively that as poll size increases, the distribution of sample proportions becomes approximately normal
- Reviewing the concept of normal distribution
- Using estimates of areas to understand the normal distribution table
- Applying the central limit theorem for the case of binomial distributions

Mean and standard deviation

- Reviewing the steps for computation of standard deviation
- Seeing that the "large number of trials" method for computing mean and standard deviation is independent of the number of trials
- Extending the concepts of mean and standard deviation from sets of data to probability distributions
- Defining the concept of variance
- Finding formulas for the mean and standard deviation of the distribution of poll results in terms of the poll size and the true proportion
- Deciding what to use for σ if the true proportion is unknown, and finding the maximum value of σ for polling problems

Confidence levels and margin of error

- Using the terminology of "confidence level," "confidence interval," and "margin of error"
- Seeing how poll size affects the standard deviation of poll results
- Establishing confidence intervals in terms of a sample proportion and the standard deviation
- Seeing how the term "margin of error" is commonly used in newspaper reporting
- Estimating the size of a poll based on its reported margin of error

